

1-1-1978

# An assessment of the ability of junior high school students to create mathematical models using computer-related curriculum materials.

Anne Dolores Pasquino  
*University of Massachusetts Amherst*

Follow this and additional works at: [https://scholarworks.umass.edu/dissertations\\_1](https://scholarworks.umass.edu/dissertations_1)

---

## Recommended Citation

Pasquino, Anne Dolores, "An assessment of the ability of junior high school students to create mathematical models using computer-related curriculum materials." (1978). *Doctoral Dissertations 1896 - February 2014*. 3430.  
[https://scholarworks.umass.edu/dissertations\\_1/3430](https://scholarworks.umass.edu/dissertations_1/3430)

This Open Access Dissertation is brought to you for free and open access by ScholarWorks@UMass Amherst. It has been accepted for inclusion in Doctoral Dissertations 1896 - February 2014 by an authorized administrator of ScholarWorks@UMass Amherst. For more information, please contact [scholarworks@library.umass.edu](mailto:scholarworks@library.umass.edu).



AN ASSESSMENT OF THE ABILITY OF JUNIOR HIGH SCHOOL  
STUDENTS TO CREATE MATHEMATICAL MODELS USING  
COMPUTER-RELATED CURRICULUM MATERIALS

A Dissertation Presented

By

ANNE DOLORES PASQUINO

Submitted to the Graduate School of the  
University of Massachusetts in partial fulfillment  
of the requirements for the degree of

DOCTOR OF EDUCATION

May 1978

Education

(c) Anne Dolores Pasquino 1978

All Rights Reserved



AN ASSESSMENT OF THE ABILITY OF JUNIOR HIGH SCHOOL  
STUDENTS TO CREATE MATHEMATICAL MODELS USING  
COMPUTER-RELATED CURRICULUM MATERIALS

A Dissertation Presented

By

ANNE DOLORES PASQUINO

Approved as to style and content by:

*William Masalski*

William Masalski, Chairperson of Committee

*Portia C. Elliott*

Portia Elliott, Member

*Alan Gleit*

Alan Gleit, Member

*Mario D. Fantini*

Mario D. Fantini, Dean  
School of Education

## A C K N O W L E D G E M E N T S

The writer wishes to express appreciation to the many persons whose assistance made this study possible.

In particular:

-- William Masalski, for his guidance, understanding and encouragement throughout this study.

-- Portia Elliott, Alan Gleit and Howard Peelle, for their many constructive criticisms as members of my dissertation committee.

-- Cynthia Walowicz and the faculty and administrators of Gateway Regional Junior-Senior High School, who assisted with the class testing of the curriculum materials.

-- Hariharan Swaminathan and Janice Gifford, at the University of Massachusetts, and Ron Edwards, Warren Hill and Harvey Roazen, my colleagues at Westfield State College, for their help and criticism during the development of this study.

-- and, my family.

ABSTRACT

An Assessment of the Ability of Junior High School  
Students to Create Mathematical Models Using  
Computer-Related Curriculum Materials

(May 1978)

Anne Dolores Pasquino, B.A., Emmanuel College  
M.A.T., Brown University  
Ed.D., University of Massachusetts

Directed by: Professor William Masalski

The purpose of this study was to develop computer-related curriculum materials which teach mathematical modeling to seventh grade students and to examine the effectiveness of these materials as a pedagogical tool for acquiring intellectual abilities and skills related to modeling. In particular, the following questions were investigated:

1. What is the relation between the student's level of abstract thinking and his/her ability to construct mathematical models?
2. How does the use of the computer-related modeling materials affect the student's ability to reason abstractly?
3. How does the use of the computer-related modeling materials with seventh grade students affect their acquisition of modeling concepts and skills?

The sample used in the study was comprised of 52 seventh grade students from Gateway Regional Junior-Senior High School in Huntington, Massachusetts. The Raven Progressive Matrices test was administered to all 52 students at the outset of the study as a measure of their level of abstract reasoning. The students were ranked according to their scores on this test; the ranked scores broken into adjacent pairs and one student from each pair randomly assigned to the treatment group with the remaining student assigned to the control group. Both the treatment and control group were also given an author-designed modeling achievement pretest at the beginning of the study.

The treatment group received instruction in mathematical modeling using the materials designed for computer implementation along with instruction in programming the computer in the BASIC language. The control group received instruction in computer programming in BASIC and were given activities on the computer which did not involve modeling. At the end of the study, the Raven Progressive Matrices test and the modeling achievement test were readministered to both the treatment and control group.

It was hypothesized that the correlation between the students' ability to think abstractly and their mathematical modeling ability would be zero. The Pearson product moment correlation coefficient computed using the Raven Progressive Matrices pretest scores and the modeling achievement pretest scores was found to be .63. A two-tailed t-test using this correlation coefficient was used to test the hypothesis that the true correlation was zero and was significant at the .05

level. The hypothesis of zero correlation was rejected which indicates that the students' level of abstract thinking was not independent of their ability to construct mathematical models.

A one-tailed t-test performed on the mean of the posttest difference scores for the experimental and control group was used to test the hypothesis that there would be no positive change in the student's level of abstract thinking as a result of exposure to the modeling curriculum materials. The t value calculated was not significant at the .05 level indicating that the students' level of abstract thinking did not increase as a result of exposure to the modeling curriculum materials.

Finally, the Wilcoxon rank-sum test was used to test the hypothesis that exposure to the computer-related curriculum materials would have no effect on the student's ability to model. The observed sum of ranks was significant at the .05 level and the hypothesis that the computer-related modeling materials have no effect was rejected.

The results of this study indicated that (1) there is a positive correlation between the student's ability to reason abstractly and his/her ability to construct mathematical models, (2) the curriculum materials did not increase the student's ability to reason abstractly, and (3) the computer-related curriculum materials developed and evaluated in this study were generally effective in teaching modeling skills and concepts to seventh grade students.

The study has significance for junior high school teaching in that it demonstrates the feasibility of teaching mathematical modeling at

this grade level and provides classroom teachers with a set of curriculum materials that have been tested and shown to be effective. On a more theoretical level, the study has significance for educational research in that it attempts to shed some light on the relationship between abstract thinking and modeling ability.



## TABLE OF CONTENTS

	<u>PAGE</u>
ACKNOWLEDGEMENTS . . . . .	iv
ABSTRACT OF THE DISSERTATION . . . . .	v
LIST OF TABLES . . . . .	xi
LIST OF FIGURES . . . . .	xii
CHAPTER	
I      INTRODUCTION . . . . .	1
Statement of the Problem . . . . .	1
Need for the Study . . . . .	2
Definitions . . . . .	5
Delimitations . . . . .	6
II      REVIEW OF THE LITERATURE . . . . .	8
Review of Curriculum Projects . . . . .	8
Review of Research in Education . . . . .	13
III      DESIGN, PROCEDURES AND MATERIALS . . . . .	22
Design of the Study . . . . .	22
Description of Subjects . . . . .	22
Assignment of Treatment Group . . . . .	23
Evaluation Instruments . . . . .	24
Null Hypotheses . . . . .	31
Procedures . . . . .	32
Procedures for Collecting Data . . . . .	32
Statistical Procedures . . . . .	33
Materials Used in the Study . . . . .	34

CHAPTER		PAGE
IV	RESULTS . . . . .	35
	Relation Between Level of Abstract Thinking and Ability to Construct Mathematical Models . . . . .	35
	Effect of Modeling Materials on Abstract Thinking . . . . .	36
	Effect of Modeling Materials on Modeling Ability . . . . .	39
V	DISCUSSION . . . . .	44
	Summary . . . . .	44
	Conclusions . . . . .	47
	Significance of the Study . . . . .	50
	Limitations of the Study . . . . .	51
	Suggestions for Further Research . . . . .	52
	BIBLIOGRAPHY . . . . .	53
	APPENDIX	
A	TAXONOMY OF MODELS . . . . .	58
B	RAVEN PROGRESSIVE MATRICES . . . . .	62
C	MODELING ACHIEVEMENT TEST . . . . .	125
D	MODELING CURRICULUM MATERIALS . . . . .	135
E	DESCRIPTION OF SAMPLE STUDENTS . . . . .	193



# LIST OF TABLES

TABLE		<u>PAGE</u>
1	PRETEST AND POSTTEST RANGE, MEAN AND STANDARD DEVIATION SCORES ON THE RAVEN PROGRESSIVE MATRICES TEST . . . . .	37
2	COMPARISON OF THE EXPERIMENTAL AND CONTROL GROUP ON POSTTEST DIFFERENCES ON RAVEN PROGRESSIVE MATRICES SCORES . . . . .	38
3	PRETEST AND POSTTEST RANGE, MEAN AND STANDARD DEVIATION SCORES ON THE MODELING ACHIEVEMENT TEST . . . . .	40
4	SUMMARY STATISTICS FOR WILCOXEN RANK-SUM TEST . . . . .	41

# LIST OF FIGURES

FIGURE		<u>PAGE</u>
1	PIAGET-PROCESS MATRIX . . . . .	19
2	SPECIFICATION CHART USED IN CONSTRUCTION OF OBJECTIVE TEST . . . . .	26
3	MODELING ACHIEVEMENT PRETEST AND POSTTEST MEANS FOR EXPERIMENTAL AND CONTROL GROUPS . . . .	43
4	RAVEN PROGRESSIVE MATRICES PRETEST AND POSTTEST MEANS FOR EXPERIMENTAL AND CONTROL GROUPS . . . . .	46

# CHAPTER I

## INTRODUCTION

### Statement of the Problem

The purpose of this study was to develop computer-related curriculum materials which teach mathematical modeling to seventh grade students and to examine the effectiveness of these materials as a pedagogical tool for acquiring intellectual abilities and skills related to modeling. The relation between abstract thinking and modeling ability was also examined. More specifically, the following questions were investigated:

1. What is the relation between the student's level of abstract thinking and his/her ability to construct mathematical models?
2. How does the use of the computer-related modeling materials affect the student's ability to reason abstractly?
3. How does the use of the computer-related modeling materials with seventh grade students affect their acquisition of modeling concepts and skills?

## Need for the Study

The formation of models has long been recognized as a powerful and effective tool in the physical sciences. Modeling is an integral part of the scientific method and has played a central role in the history of the development of new theories in science. The success of the modeling process in biology, chemistry and physics need not be chronicled here. Suffice it to say that this success has, in recent years, become the impetus for social scientists to join mathematicians in applying modeling techniques to problems in the behavioral sciences.

Educators and curriculum developers have also come to recognize the importance of the modeling process as a problem solving strategy. The Cambridge Conference on the Correlation of Science and Mathematics in the Schools (1969) addressed the question of how an acquaintance with the scientific method might improve decisions that an individual must make to function effectively in society. The conferees held that:

The way in which observation and reason are brought to bear on everyday as well as technical problems is through the vital process of constructing simplified conceptual models for real-world objects and interactions (p. 16).

Major middle school (grades five through eight) science curriculum improvement projects developed in the 1960's and early 1970's such as science--A Process Approach (SAPA), Science Curriculum Improvement Study (SCIS), and Unified Science and Mathematics for Elementary Schools (USMES)--all voice the recommendation that students be exposed

to the process of model building as part of the scientific method. More recently, the 1975 National Advisory Committee on Mathematical Education report, Overview and Analysis of School Mathematics, Grades K-12, also pointed to the need for "instructional materials at all levels in: the use of . . . applications and modeling, . . . and the general ability to collect, interpret, and understand quantitative information, . . ." (p. 145).

In addition to being a valuable process skill which can be consciously cultivated for problem solving, the construction of models plays an important role in intellectual development. Several psychologists have written about the importance of mental models in teaching and learning. For example, Bruner (1962) relates model development to mental development.

I am inclined to think of mental development as involving the construction of a model of the world in the child's head, an internalized set of structures for representing the world around him. These structures are organized in terms of perfectly definite grammars or rules of their own, and in the course of development the structures change and the grammar that governs them also changes in certain systematic ways. The way in which we gain lead time for anticipating what will happen next and what to do about it is to spin our internal models just a bit faster than the world goes (p. 103).

Piaget (Ginzburg and Oppen, 1969) speaks of a child's "representation" or "schema" which is a mental or conceptual model the child has built within his/her mind and which forms the basis for all interactions with the real world. The model or schema is not static, but is constantly being updated by accommodation and assimilation as new information is received. Minsky (1968, 1970) has expressed the same idea and talks

about building, testing and debugging internal (mental) models of reality.

Suchman (1964) has reported from his studies in Inquiry Training that models are a part of the inquiry cycle.

Two kinds of scanning are used: scanning the field for data, and scanning the store of ideas for conceptual models. These comprise two parts of the inquiry cycle. A child absorbs a percept and he tries to find a model he can use to assimilate it. Then, providing assimilation is incomplete, he performs some action to generate new data. At the same time that he acquires new data, he scans for new models on which to test the data. He endeavors to match the data coming in with the models being tried out. At some point, the match between the data and the model is made (p. 230).

In spite of the recognition afforded modeling by physical scientists, psychologists, educators and curriculum specialists, mathematical modeling and modeling in general does not occupy a prominent position in junior high school mathematics and science curricula today. With the exception of the USMES materials, there appears to be no concentrated effort on the part of curriculum developers to include mathematical models and mathematical model building in the junior high school science and/or mathematics-science curricula. The absence of curriculum materials which teach mathematical modeling is puzzling in light of the foregoing discussion. Science curriculum projects stress the desirability of exposing students to the processes of the "scientific method"; yet mathematical modeling, an integral part of the scientific method, is not taught in the actual curriculum materials.

One can begin to understand why mathematical modeling does not occupy a more prominent position in science and mathematics curricula



by looking at how developmental psychologists and learning theorists conceive of the child's ability to handle the abstractions involved in the construction of mathematical models. The research of Piaget indicates that children at this age level (grade seven) do not in general possess the ability to hypothesize and to reason logically which are skills necessary to construct mathematical models. On the other hand, Gagne, in The Conditions of Learning (1965), states that the ability of the child to perform a task (such as learning to construct mathematical models) is essentially a function of the presence or absence of prerequisite learning. (It should be noted, however, that Gagne, in a recent edition of this book, indicates that he has modified his position somewhat and allows that the various forms of prerequisite learning may, in fact, be attained in Piagetian-like stages.) Since very little research has been done in this area, there is a need for empirical studies which will shed some light on the question of whether or not junior high school children possess the ability to construct mathematical models.

### Definitions

For the purposes of this study, the following terms are defined:

#### Computer Program

A program is an ordered sequence of formal statements, usually expressed in a programming language suitable for execution by a computer.

### Computer-Related Modeling Materials

Computer-related modeling materials refer to a particular set of instructional materials designed by the author to teach mathematical modeling to seventh grade students. These materials are attached in Appendix D.

### Mathematical Model

A mathematical model is defined as "the representation of a physical phenomenon or a real-world situation by some mathematical entity; for example, by numbers, an equation or a system of equations, a function, an algorithm or a structure such as a group" (Bell, 1971, p. 293). In this study, the researcher is concerned with the development of simple quantitative mathematical models and, in most instances, the computer models developed to represent physical phenomena will require only elementary arithmetic.

A general overview of the various categories of models has been developed by the author in the form of a taxonomy and can be found in Appendix A.

### Level of Abstract Thinking

Level of abstract thinking is operationally defined to be measured by performance on the Raven Progressive Matrices test.

### Delimitations

This research was limited to an examination of the effectiveness of specific computer-related curriculum materials as a learning tool.



The study was limited to two classes of seventh grade students from Gateway Regional Junior-Senior High School, Huntington, Massachusetts. Both classes were taught by the same teacher, Mrs. Cynthia Walowicz, who co-authored the curriculum materials. This limited sample and teacher characteristics impose the usual curtailments relative to the generalizability of the study. Also, the study has theoretical limitations in that "level of abstract thinking" has been operationally defined as performance on the Raven Progressive Matrices test.

Given these limitations, the study represents a necessary first endeavor to resolve questions concerning the relation between abstract thinking and modeling ability and the acquisition of modeling skills at the junior high school level.

## CHAPTER II

### REVIEW OF THE LITERATURE

A review of the research related to the teaching of mathematical modeling via the computer can be divided into two parts. The first part deals with a review of major integrated mathematics-science curriculum projects and science curriculum projects that have been tested and implemented in the middle school (grades five through eight) over the past fifteen years and a review of those curriculum improvement projects which involve the use of the computer as an instructional tool controlled by the student. The second part reviews the research done by developmental psychologists, learning theorists and educators concerning the ability of children to construct conceptual models.

#### Review of Curriculum Projects

A review of mathematics projects of the 1960's and 1970's reveal that such projects emphasized the teaching of the structure of mathematics and mathematical concepts rather than applications. In general, the teaching of mathematical modeling did not play a significant role in these projects. Therefore, a review of integrated mathematics-science and science curriculum projects in grades five through eight was subsequently undertaken and proved more fruitful. The projects

reported here are those characterized by two or more of the following features. The project:

- 1) stressed the integration of mathematics and science;
- 2) stressed the process approach, i.e., focused on skills needed to acquire, organize, generate and utilize information;
- 3) recognized the development of modeling skills as a major or minor project goal; or
- 4) could be considered a major project in terms of having reached a wide audience or having attained national reputation.

Among others, the projects reviewed included: the Minnesota Mathematics and Science Teaching Project (MINNEMAST), Elementary School Science (ESS), Conceptually Oriented Program in Elementary Science (COPES), Science Curriculum Improvement Study (SCIS), Science--A Process Approach (SAPA), and Unified Science and Mathematics for Elementary Schools (USMES).

Curriculum development groups began to focus attention on science in the elementary and middle school curriculum in the late 1950's and early 1960's. Unlike mathematics which had been in the elementary and middle school for generations, science was not present in any significant degree in these grade levels at that time (Goals for the Correlation of Elementary Science and Mathematics, 1969, p. 28). Science curriculum materials were developed from about 1960 to 1970 but have

had "a slow adoption rate in the elementary and middle schools" (Paldy, 1972, p. 576). (See also MACE report, Essentially Elementary Science, 1973.)

Very few curriculum development projects attempted to present a truly integrated approach to the teaching of mathematics and science. MINNEMAST and USMES are two projects which did attempt an integrated approach. The MINNEMAST materials were originally intended to cover grades K through eight, but only materials for grades K through three were developed. The USMES materials on the other hand have evolved to a point where they now integrate the teaching of the social sciences as well as mathematics and science in grades one through eight.

The type of model building that takes place in the MINNEMAST materials is the construction of physical models or scaled down replicas of existing objects. Hence, the type of model used is not a mathematical model as defined in this study (see definitions in Chapter I). The original intent of the USMES materials, as specified by the Cambridge Conference on the Correlation of Science and Mathematics in the Schools, was to teach children to develop simplified conceptual models of real-world objects and interactions. Building the conceptual model would require the abstraction of certain entities and relations, which are often numeric and algebraic, from the real-world. The ability to manipulate or calculate the model entities would permit the child to predict and control the behavior of real objects. The units on real-world problems as developed by the

USMES Planning Committee tend, however, to be very task oriented and there is no strong evidence to indicate that children do develop conceptual models in order to complete the tasks. While much computation is involved in solving the real problems or challenges, one is often hard pressed to find the type of mathematical model described in the Cambridge Conference report.

The SCIS, SAPA and ESS science curriculum materials all stress the importance of exposing the child to the scientific method of inquiry. The materials speak of scientific models and mental models, but they are not referring to physical or mathematical models. Rather, they have in mind a scientific theory or possible explanation of how a system functions when they use the word model. To the authors of these materials, a scientific model is "a mental image of a real system to which you assign certain parts or properties that you cannot see directly" (SCIS Sample Guide, 1970, p. 8).

Curriculum improvement projects which stress student control of the computer were also reviewed in an attempt to learn what age levels had successfully utilized the computer as an instructional tool and to see what (if any) computer curriculum materials had been developed to teach mathematical modeling. Traditional computer-assisted or computer-managed instruction projects (e.g., the PLATO IV System), where the computer controls the interaction with the students, were not included in this review.

The computer projects examined included: the LOGO Project, the Computer Assisted Mathematics Program (CAMP), Project Solo, the



Huntington Project, and the Hewlett-Packard Curriculum Project. A result of the LOGO Project, which has significance for this study, is the fact that it is possible to teach programming to "average" seventh grade, and younger, children. The LOGO programming language has been used successfully by second and third graders.

The CAMP program consists of a series of six textbooks for grades seven through twelve. The purpose of these texts is to acquaint students with the problem solving aspects of the computer in their regular school work. The materials use the BASIC programming language and stress use of the computer as a tool to assist in the learning of skills, concepts and problem solving in mathematics. CAMP has the distinction of being one of the first attempts (1965-1967) to implement learner control of the computer rather than use the computer as a tutor.

Both the Huntington Project and the Hewlett-Packard Curriculum Project produced computer simulations in the physical and social sciences, but these materials are aimed at the high school and freshman college audiences. The student using the Huntington Project materials is allowed to specify certain variables in the simulation and in this way affect the outcome of the program. However, the simulations are written principally to teach content and not simulation. Hence, the student is not expected or required to modify the program, but rather to use it as is. Hewlett-Packard has published one curriculum book in environmental science which is designed to allow the student to write simulation programs given some initial information

and a beginning model. This publication is unique in that its intent is to teach computer modeling techniques by having the student build and refine programs that model the effects of air pollution.

No computer curriculum improvement projects were found which attempted to teach computer modeling techniques to students at the middle school level.

In summary, the review of curriculum projects indicates that while the major mathematics-science and science curriculum projects express the desirability of teaching children to model, the teaching of mathematical modeling per se is not included in the curriculum project materials. Neither are there presently available computer-related curriculum materials which attempt to teach mathematical modeling at the junior high school level.

### Review of Research in Education

The second part of the literature search is a review of research done by developmental psychologists, learning theorists and educators concerning the ability of children to construct conceptual models.

One of the major influences on science and mathematics curricula in recent years has been the developmental theory of Jean Piaget. The extensive investigations of Piaget have led him to describe four major stages of intellectual development. Only in the last stage, called the stage of formal operations, does the individual reason about the relationships and implications of hypotheses as well as actualities. In the next-to-the-last stage, that of concrete operations, the person

is able to reason by using operations, such as classification, serial ordering, and time sequencing, on objects, but cannot yet apply such operations to verbally expressed hypotheses. Preceding this is a stage that is pre-operational. In this stage, objects exist and their present appearances can be described by the child, but changes with time or as a result of physical transformations are not comprehended. During infancy, there is a sensory-motor, preverbal stage during which the permanent existence of objects and simple spatial relations are established through a combination of visual and kinesthetic explorations (Karplus, 1972).

To see what implications Piaget's "stages" hold for mathematical modeling, we consider one of his investigations in some detail. In this investigation, the subjects were required to solve what is essentially a problem in physics: to discover which of four factors, length, weight, height or force, alone or in combination with the others, affects the frequency of oscillation of a simple pendulum. The subject was allowed to experiment with the pendulum in any way he/she pleased. Pre-operational children approached the task in a very haphazard way. The "experiments" which they devised revealed no overall plan or pattern. They seemed to make random tests which yielded little information of value. The child's expectations influenced observations and the conclusions reached were faulty and unrelated to evidence.

The concrete operational child shows considerable improvement in intellectual ability. The child examines a number of potential



determinants of oscillation and observes results in an accurate way, but fails to design experiments which generate sufficient information to decide among several alternatives and one alternative is chosen without sufficient justification.

In the stage of formal operations, the child performs well in all three aspects of the problem. He/she designs experiments properly, observes results accurately and draws the proper logical conclusions from the observations.

At the formal operational stage, the child has the ability to consider all the possible combinations of variables before beginning the experiment. The concrete operational child does not do this and is limited to dealing with empirical results, with things that are available to immediate perception. Furthermore, in the formal operational stage, experiments are based on deductions from the hypothetical rather than being bound solely by what was observed. Formal operational thought, unlike the thought of the concrete operational child, is hypothetico-deductive.

The results of this investigation and the learning theory of Piaget indicate that children will be successful in handling mathematical models only after they have reached the formal operational stage. Piaget places the age at which children usually reach the stage of formal operations at somewhere between eleven and sixteen years.

In contrast to Piaget, another influential learning theorist, Robert Gagne, has maintained that the ability of the child to handle conceptual models is essentially a function of the presence or

absence of prerequisite learning. The SAPA materials have been strongly influenced by the psychological theories of Gagne who emphasizes the hierarchical development of science process skills.

Jerome Bruner, in discussing readiness for learning in The Process of Education (1960), put forth his oft quoted hypothesis that "any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (p. 33). Bruner recognizes the developmental stages of Piaget, but rather than view them as placing limitations on topics of instruction, he views the stages as a mandate to teachers to present and structure information in a form which can be understood by the child at his/her present developmental stage. He also believes that instruction can lead intellectual development by providing "challenging but usable opportunities" for the child to move ahead into the next stages of development. Hence, Bruner and Gagne have traditionally been interpreted as viewing the child's ability to learn as essentially a problem in the presentation and structuring of information by the teacher rather than a problem of the child's developmental level.

A review of current research in education indicates that there has been very little empirical research done on the ability of children to formulate mathematical models. Existing research focuses upon the ability of elementary school children to formulate conceptual models to explain their observations of natural phenomena, and related research can be found which deals with the acquisition of the logical operations which Piaget deems necessary for model building and

hypothesizing.

A study by Liao (1972) investigated the use of analog computer simulation as a pedagogical tool for acquiring modeling concepts and skills. The study was done in connection with the Engineering Concepts Curriculum Project (ECCP), which developed materials for a laboratory science course to be used by students in the last three years of high school. The results of the study indicated that providing students with modeling activities helped them improve their modeling skills and that use of the analog computer encouraged students to use more precise or quantitative methods of describing information.

Anderson (1965) reports on the ability of children (grades three through six) to formulate conceptual models to explain certain demonstrations of natural phenomena. For example, one such demonstration involved the phenomena of surface tension in water. The type of model required in this situation was a "plausible explanation" which did not necessarily possess quantitative components. Anderson found that the ability to formulate conceptual models increased with age and IQ level. Also, the consistency of explanations increased with age and the frequency of atomistic models increased with age and IQ level.

Anderson's work suggested to Ziegler (1967) that children might be characterized as modelers and non-modelers according to the types of explanations they used. Ziegler's study showed that not only could students be classified, but students' use of models could be enhanced by instruction. McIntyre (1972) extended the work of Anderson and

Ziegler which concentrated on the area of the particle nature of matter to include phenomena associated with electrostatics. His study confirmed and extended the idea that elementary school children can and do use models or physical analogies in their explanations of physical phenomena. Grade levels two through six were used, and it was found that students in grades five and six tended to use analogies in their explanations more frequently than students in grades two through four. It was suggested that the modeling on the part of the older students was probably the result of instruction and that the effectiveness of various types of models might be a suitable topic for further study.

In addition to studies which deal directly with children's ability to model, related research discusses logical operations which are relevant for modeling. Wood (1974) has developed a Piaget-Process Matrix (see Figure 1) which he uses to visualize the intersection of the processes of science with Piaget's developmental stages. The horizontal axis of the matrix indicates the various stages of intellectual development while the vertical axis indicates the process of science under study. Each cell in the matrix results in one of three possible educational implications. The implications are: (1) the process is not appropriate for the stage of development, (2) the process is appropriate for the stage of development, or (3) sufficient evidence for making a determination does not exist.

The matrix has significance for this study in that it indicates that three processes, which are important in modeling, namely,

FIGURE 1

## PIAGET-PROCESS MATRIX

Process of Science	Sensorimotor	Preoperational	Concrete Operational	Formal Operational
Observation				
Qualitative	X	X	X	X
Quantitative			X	X
Classification				
Single Attribute		X	X	X
Multiple Attribute			X	X
Controlling Variables				
Single Variation			X	X
Multiple Variation				X
Generalization				
Induction				
Formulating Hypotheses			X	X
Creativity				
Testing Hypotheses				
Deduction				X

(X) Appropriate level for teaching the process.  
 (Blank) Inappropriate level for teaching the process.  
 (-) Insufficient research.

Note: From "The Piaget-Process Matrix" by D.A. Wood, School Science and Mathematics, 1974, Vol. 74, pp. 407-412.



controlling multiple variation, formulating hypotheses and testing hypotheses, are either inappropriate for children at the concrete operational stage or, as in the case of formulating hypotheses, there is insufficient research available for making a determination.

Raven (1974) has done research on facilitating Piaget's logical operations through programmed instruction. He investigated the effect of training on the classification operations, compensatory operations and the logical operations necessary for correlational thinking. The use of structured training materials was found to enhance the achievement of pupils on logical operations tasks. His work on the classification operations was done with second and third grade children, the work on compensatory operations involved sixth and eighth graders and the work on logical operations was done with fifth, seventh and ninth grade students.

Other than the work of Liao, which deals with high school students, no literature was found by this researcher which dealt with the ability of children to formulate mathematical models.

In summary, the developmental stages of Piaget indicate that some aspects of mathematical modeling (e.g., testing hypotheses) may be beyond the reach of middle school students who are at the concrete operational stage (see Wood's Piaget-Process Matrix). Yet, there is evidence that shows children in grades two through six (particularly grades five and six) can and do use conceptual models and that their use of conceptual models improves with instruction. Furthermore, there is evidence (Raven, 1974) that the underlying logical operations

necessary for modeling can be facilitated by instruction. Hence, there are many unanswered questions concerning the feasibility of teaching mathematical modeling to middle school students whose level of thinking may be at the concrete operational stage or at the transitional stage between concrete and formal operations. This study was designed, therefore, to investigate questions regarding the relationship between the level of abstract thinking of seventh grade students and their ability to construct mathematical models.

## CHAPTER III

### DESIGN, PROCEDURES AND MATERIALS

This chapter contains a discussion of the design of the study (treatment, subjects, evaluation and null hypotheses), procedures for collecting and analyzing the data, and materials used in the study.

#### Design of the Study

##### Description of the Subjects

A sample of 52 students (28 males and 24 females) was used in the study. All were seventh grade students attending Gateway Regional Junior-Senior High School in Huntington, Massachusetts. Gateway Regional Junior-Senior High School is located in a rural school district incorporating seven communities. The students represented a wide range of socioeconomic backgrounds. Most of the students had a "B" or "C" grade average, and estimated WISC IQ scores obtained from recently administered Academic Promise Tests (APT) indicated that the sample was "average" in mental ability. A detailed description of each student can be found in Appendix E. The description includes the student's age, pretest and posttest Raven Progressive Matrices test scores, pretest and posttest modeling achievement scores, the Abstract Reasoning subtest score from the Academic Promise Tests, and an



estimated WISC IQ score which was predicted by the total score on the Academic Promise Tests.

#### Assignment of Treatment Group

All 52 students involved in the study were given the Raven Progressive Matrices test (see Appendix B) as a pretest and were ranked according to their percentile score on this test. The ranked percentile scores were broken into adjacent pairs and one student from each pair randomly assigned to the treatment group and the remaining student assigned to the control group. Both the treatment and control group were given a modeling achievement pretest (see Appendix C) prior to the beginning of the study.

The treatment group received instruction in mathematical modeling using curriculum materials designed for computer implementation (see Appendix D) along with instruction in programming the computer using the BASIC language. The control group also received instruction in computer programming in BASIC and were given activities on the computer which related to their regular syllabus but which did not involve modeling. (BASIC was chosen as the programming language for this study, since it could be taught to seventh graders in a relatively short time and because of the availability of a BASIC compiler at Gateway Regional Junior-Senior High School.)

At the end of the study, the Raven Progressive Matrices test and the modeling achievement test were readministered to both the treatment and control group.

### Evaluation Instruments

Two evaluation instruments were used in the study: the Raven Progressive Matrices test and a modeling achievement test constructed by the author.

The Raven Progressive Matrices test was used as a measure of abstract reasoning ability (see Definitions). Finding a group test of abstract reasoning ability which was suitable for seventh grade students proved to be a difficult task due to the scarcity of available instruments. The Raven Progressive Matrices test was selected from the few commercially available group tests. The test consists of 60 matrices, or designs, from each of which a part has been removed. The test taker chooses the missing part from six or eight given alternatives. "An answer which fits may: (a) complete a pattern, (b) complete an analogy, (c) systematically alter a pattern, (d) introduce systematic permutations, or (e) systematically resolve figures into parts. The number of items correctly solved is the score which is then translated into a percentile rank" (Buros, 1965, p. 491). Many studies have been published which provide relevant data on this test (Burke, 1958). These studies indicate that the test has fair concurrent validity and a retest reliability that varies between .70 and .90. The Progressive Matrices test yields a purely nonverbal score which does not penalize those students who have good reasoning ability but who are below standard in reading and verbal development.

The 52 students involved in this study were administered a battery of tests called the Academic Promise Tests during the academic

year 1976-1977, as part of the guidance testing program at Gateway Regional Junior-Senior High School. A correlation coefficient of .67 was computed between the Raven Progressive Matrices pretest scores of these students and the Abstract Reasoning subtest scores from the Academic Promise Tests.

Since there was no available evaluation instrument for measuring modeling achievement which was directly suited to this study, the author had to develop her own instrument. A specification chart (see Figure 2) listing the major content areas (phases of modeling), and educational objectives (intellectual abilities and skills related to modeling) was used to prepare the questions for the modeling achievement test. The details of construction of this chart and the modeling achievement test are discussed in a later section.

The modeling achievement test, which was used as a pretest and posttest in this study, was an objective test consisting of twenty questions. Eighteen of the twenty questions were multiple choice questions and two questions required that the student write a short computer program in BASIC. The objective test was revised based on feedback from a trial group. After revision, the reliability of the test was estimated by administering it to 20 seventh grade students who were not involved in the study. Using the split-half method, a reliability coefficient of .83 was obtained. Since the test was designed to measure achievement, it was analyzed for content validity. In other words, how well do the tasks of the test represent what experts consider to be important outcomes in this area of instruction? The

FIGURE 2  
SPECIFICATION CHART USED IN CONSTRUCTING OBJECTIVE TEST

FIGURE 2

SPECIFICATION CHART USED IN CONSTRUCTING OBJECTIVE TEST

MAJOR CONTENT AREAS (Phases of Modeling)	EDUCATIONAL OBJECTIVES (Intellectual Abilities and Skills Related to Modeling)	TOTAL						
	1. Achieve knowledge and understanding of the modeling process as evidenced by:		2. Achieve skill in constructing models as evidenced by:					
			Comprehension			Application	Revision	
	Working Vocabulary	Factual Knowledge of Methodology & Procedures	Trans-lation	Inter-pretation	Extrap-olation			
Concept of a Model (Uses, Advantages, Limitations)	5%	10%					15%	
Steps Involved in the Modeling Process	5%	10%					15%	
Model Building (Pattern Recognition, Expression, Hypothesizing)			15%	15%	10%	15%	62%	
Model Evaluation (Testing, Revision)							8%	
TOTAL	10%	20%	15%	15%	10%	15%	100%	

major content areas and educational objectives used in the specification chart reflect the goals and content of the curriculum materials used in the study. The content areas and educational objectives together with their relative weight were agreed upon by a panel of educators with experience in the teaching of modeling and/or seventh grade curriculum. The twenty objective questions are representative of instruction in modeling because each question tests for a specific "agreed-upon" outcome, and each cell of the specification chart is represented by a specific percentage of questions.

Specification Chart and Modeling Achievement Test. The modeling achievement test was constructed using the specification chart shown in Figure 2.

The numbers in each of the cells in the chart indicate the percentage of test items devoted to each area of content and each type of objective. For example, with regard to the content area of "Steps in the Modeling Process," 5 percent of the items in the test were concerned with knowledge of "working vocabulary," and 10 percent were concerned with "factual knowledge of methodology and processes." Empty cells indicate that there were no items allotted to these areas.

The relative weight given to each area of content and each type of objective is indicated by the total percentage of items in a given category. For example, the right-hand column of Figure 2 shows that 15 percent of the items are concerned with the "Concept of a Model," 15 percent with "Steps in the Modeling Process," and so on down the



column. Similarly, the bottom row of the chart shows that 10 percent of the items are devoted to "working vocabulary," 20 percent to "factual knowledge of methodology and procedures," and so on across the row.

The vertical categories in the specification chart are four major content areas or bodies of knowledge which were taught via the curriculum materials.

1. The concept of a model involves knowing what a model is: its purpose and uses, advantages and limitations.
2. The steps involved in the modeling process refers to factual knowledge of the following steps:
  - a) Measurement and observation of some real world phenomenon.
  - b) Identification of relevant variables, simplification of data, recognition of patterns and/or relations in the data.
  - c) Step (b) leads to the formation of hypotheses and the construction of a model or description.
  - d) Predictions and data generated from the model are evaluated by comparing them to additional data from the real world.
  - e) If the evaluation in step (d) shows that the model is unsatisfactory, then the model is refined or revised and step (d) is repeated.
3. Model building involves the student's implementation of the steps in the modeling process. That is, the student actually performs the following tasks:



measurement or data collection, isolation of relevant variables, simplification of data, pattern recognition, expression of relations and/or patterns and hypothesizing.

4. Model evaluation involves the student's testing and revising the model.

The horizontal categories are knowledge of terminology and methodology and intellectual abilities and skills related to modeling. They follow some of the categories discussed in Taxonomy of Educational Objectives (Bloom, 1956) with the addition of a Revision category developed by Liao (1971). The following are brief descriptions of each category:

1. Knowledge -- The remembering of previously learned material. In this case it involves the recall of terminology and of methodology or procedure.

Educational Objective: The ability to define various modeling terms and to describe the methodology of modeling.

2. Comprehension -- An understanding of the literal message contained in a communication.

- a) Translation -- An individual can put a communication into other language, into other terms and into another form of communication.

Educational Objective: The ability to transform communication from one form to another form (e.g., to translate numerical data into a computer program which can reproduce the data).

- b) Interpretation -- The act of dealing with a communication as a configuration of ideas.

Educational Objective: The ability to recognize or make inferences which may be drawn from the communication.

- c) Extrapolation -- The making of estimates or predictions based on understanding the trends, tendencies or conditions described in the communications.

Educational Objective: Ability to:

- 1) Predict continuation of trends.
- 2) Estimate or predict consequences of courses of actions described in a communication.

- 3. Application -- The use of models in particular concrete situations.

Educational Objective: The ability to identify and use generalizations to solve actual problems.

- 4. Revision -- Modifying or changing existing models based on the following:

- a) More accurate data (conflict of new data with original model).

- b) Changes in the system being modeled.

Educational Objective: The ability to modify existing models via analysis and evaluation of the existing model. This basically involves the synthesis of an improved model.

"The revision category involves the last three skill categories of Bloom's Taxonomy of Educational Objectives: Analysis, Synthesis and Evaluation. Revision of an existing model of a system involves analyzing different elements of the model and comparing it to information obtained from the system (evaluation based on external evidence). As a consequence of this analysis and evaluation, a new model may have to be synthesized which will be more representative of the system

being modeled. The revision phase of modeling is an essential component of the modeling process" (Liao, 1971, p. 19).

### Null Hypotheses

Based upon the researchable questions presented in Chapter I, the following null hypotheses were used in this study:

1. The correlation between the student's ability to think abstractly and his/her mathematical modeling ability will be zero. (The student's modeling ability and ability to think abstractly are independent.)
2. The mean of the difference scores on the Raven Progressive Matrices posttests for the experimental and control group will be less than or equal to zero. (Exposure to the computer-related modeling materials will not have a positive effect on the student's ability to think abstractly.)
3. There will be no significant difference between the distributions of the pretest-posttest difference scores of modeling achievement for the experimental and control groups. (Exposure to the computer-related modeling materials will have no effect on the student's ability to create mathematical models.)

The alternative hypotheses respectively were:

1. The correlation between the student's ability to think abstractly and his/her mathematical modeling ability will not be zero. (The student's modeling ability and ability to think abstractly are not independent.)
2. The mean of the Raven Progressive Matrices posttest scores for the experimental group will be greater than the mean of the Raven Progressive Matrices posttest scores for the control group. (Exposure to the computer-related modeling materials will have a positive effect on the student's ability to think abstractly.)
3. The average value of the modeling achievement scores for the experimental group will be greater than the average value of the modeling achievement scores for the control group. (Exposure to the computer-related modeling materials will have a positive effect on the student's ability to create mathematical models.)

## Procedures

### Procedures for Collecting Data

The students in both the control and treatment group were taught by their regular classroom teacher, Mrs. Cynthia Walowicz, for a period

of five weeks. She met with them for five sessions per week, each session lasting 50 to 75 minutes (scheduled class periods varied from day to day). In presenting the modeling materials to the treatment group, she followed the guidelines outlined in the Handbook for Teachers (see Appendix D). All the students (treatment and control groups) were given the modeling achievement test as a pretest and as a posttest. The Raven Progressive Matrices Test was also given as a pretest and as a posttest to both groups.

### Statistical Procedures

The Pearson product moment correlation coefficient was calculated using the pretest Progressive Matrices scores and the pretest modeling achievement scores for the entire sample of 52 students. A two-tailed t-test was used on the correlation coefficient to test Hypothesis 1, which stated that the true correlation was zero.

Hypothesis 2 was tested by performing a one-tailed t-test on the mean of the difference scores on the Progressive Matrices posttests for the control and experimental group.

Hypothesis 3 was tested by using the Wilcoxon rank-sum test (also known as the Wilcoxon T test or the Mann-Whitney U test) on the pretest-posttest modeling achievement difference scores for the control and experimental group. A one-tailed test was performed.

A significance level of .05 was used for all tests.



### Materials Used in the Study

As discussed earlier, two evaluation instruments were used in the study, the Raven Progressive Matrices Test and the modeling achievement test. Computer-related modeling materials for the treatment group were developed specifically for this study. The materials were developed in conjunction with junior high school teachers to insure that the materials modeled concepts which are appropriate for students at this level. The materials consisted of three units and were an adaptation of some of the models found in The Limits to Growth (Meadows, et al., 1971). Unit I dealt with linear and exponential growth models and their application to population growth and GNP growth for various countries. Unit II discussed the use of exponential growth models in predicting the lifetime of various finite natural resources. Finally, Unit III was designed to mitigate the Malthusian flavor of the first two units by suggesting that technology grows exponentially. The complete set of curriculum materials is contained in Appendix D.

To implement the design of the study, it was necessary that both the treatment and control group have the use of a digital computer. Gateway Regional Junior-Senior High School has an in-house PDP-8/E computer manufactured by Digital Equipment Corporation in Maynard, Massachusetts. The PDP-8/E has 8 K of memory with a BASIC compiler. The students had access throughout the study to two ASR Model 33 teletypes and a Decwriter terminal.



## C H A P T E R   I V

### RESULTS

Upon completion of the collection of data, the null hypotheses stated in Chapter III were tested using the statistical tests outlined in the Procedure section. The quantitative results of this statistical analysis are reported in this chapter.

#### Relation Between Level of Abstract Thinking and Ability to Construct Mathematical Models

It was hypothesized that the correlation between the students' ability to think abstractly and their mathematical modeling ability would be zero. To test this hypothesis, the Raven Progressive Matrices test and the modeling achievement test were administered to the entire sample of 52 students at the beginning of the study. The Pearson product moment correlation coefficient was computed using the Raven Progressive Matrices pretest scores and the modeling achievement pretest scores and was found to have a value of .63.

After a chi-square goodness-of-fit test was performed to verify the normality of the modeling achievement test scores, a two-tailed t-test was used to test the hypothesis that the true correlation was zero. A value of  $t = 5.73$  was computed which is significant at the

.05 level. The 95 percent confidence interval around an  $r$  of .63 extends from .431 to .770. That is, the probability is .95 that the interval (.431; .770) covers the true value of  $\rho$ , the population correlation coefficient. Hence, the hypothesis of zero correlation can be rejected. The students' level of abstract thinking as measured by the Progressive Matrices test is not independent of their ability to construct mathematical models.

#### Effect of Modeling Materials on Abstract Thinking

The second null hypothesis stated that there would be no positive change in the student's level of abstract thinking as a result of exposure to the modeling curriculum materials. At the beginning of the study, the experimental and control group subjects were matched on the basis of their Progressive Matrices scores. Table 1 indicates that the means of the pretest scores were almost identical with mean scores of 55.85 and 55.31 for the experimental and control groups, respectively.

Upon completion of the study, the Progressive Matrices test was readministered and a one-tailed  $t$ -test performed on the mean of the posttest difference scores for the experimental and control group (Table 2). A value of  $t = -0.10$  was found which is not significant at the .05 level. Hence, the hypothesis that the mean of the Raven Progressive Matrices posttest difference scores for the experimental and control group would be less than or equal to zero, cannot be rejected. That is, the students' level of abstract thinking did not

TABLE 1  
PRETEST AND POSTTEST RANGE, MEAN, AND STANDARD DEVIATION  
OF SCORES ON THE RAVEN PROGRESSIVE MATRICES TEST

Group	N	Pretest			Posttest		
		Range	M	s	Range	M	s
Experiment	26	18-98	55.85	23.25	24-96	59.00	23.40
Control	26	15-96	55.31	22.67	18-96	59.42	20.56

TABLE 2  
COMPARISON OF THE EXPERIMENTAL AND CONTROL GROUPS ON POSTTEST DIFFERENCES  
ON RAVEN PROGRESSIVE MATRICES SCORES

N	df	$M_D$	$s_D$	est $\sigma_{M_D}$	t
26	25	-0.42	20.39	4.08	-0.10

increase as a result of exposure to the modeling curriculum materials.

### Effect of Modeling Materials on Modeling Ability

Hypothesis 3 stated that exposure to the computer-related curriculum materials would have no effect on the student's ability to model. The pretest and posttest range, mean and standard deviation of scores on the modeling achievement test are presented in Table 3.

Since the experimenter was not prepared to rule out non-normal distributions for the modeling achievement pretest-posttest difference scores, a nonparametric or distribution-free test was used to study Hypothesis 3. This choice also seems appropriate since the control and experimental groups were not matched on their pretest modeling achievement scores and the difference scores might not be independent of the pretest scores. It should be noted that in the event that the distribution of the difference scores is approximately normal, then the distribution-free Wilcoxon rank-sum test compares quite well with the t-test as regards efficiency and power. When nothing is known about the shape of the distribution, then the Wilcoxon test may be considerably more efficient than the t-test.

The Wilcoxon rank-sum test was used to test the hypothesis that exposure to the computer-related modeling materials would not affect the student's ability to model. The summary statistics for the Wilcoxon test are reported in Table 4.

TABLE 3

PRETEST AND POSTTEST RANGE, MEAN AND STANDARD DEVIATION OF SCORES ON THE MODELING ACHIEVEMENT TEST

Group	N	Number of Possible Points	Pretest		Posttest	
			Range	M	Range	M
Experiment	26	100	2-63	32.27	24-89	54.58
Control	26	100	1-63	26.69	8-76	34.77



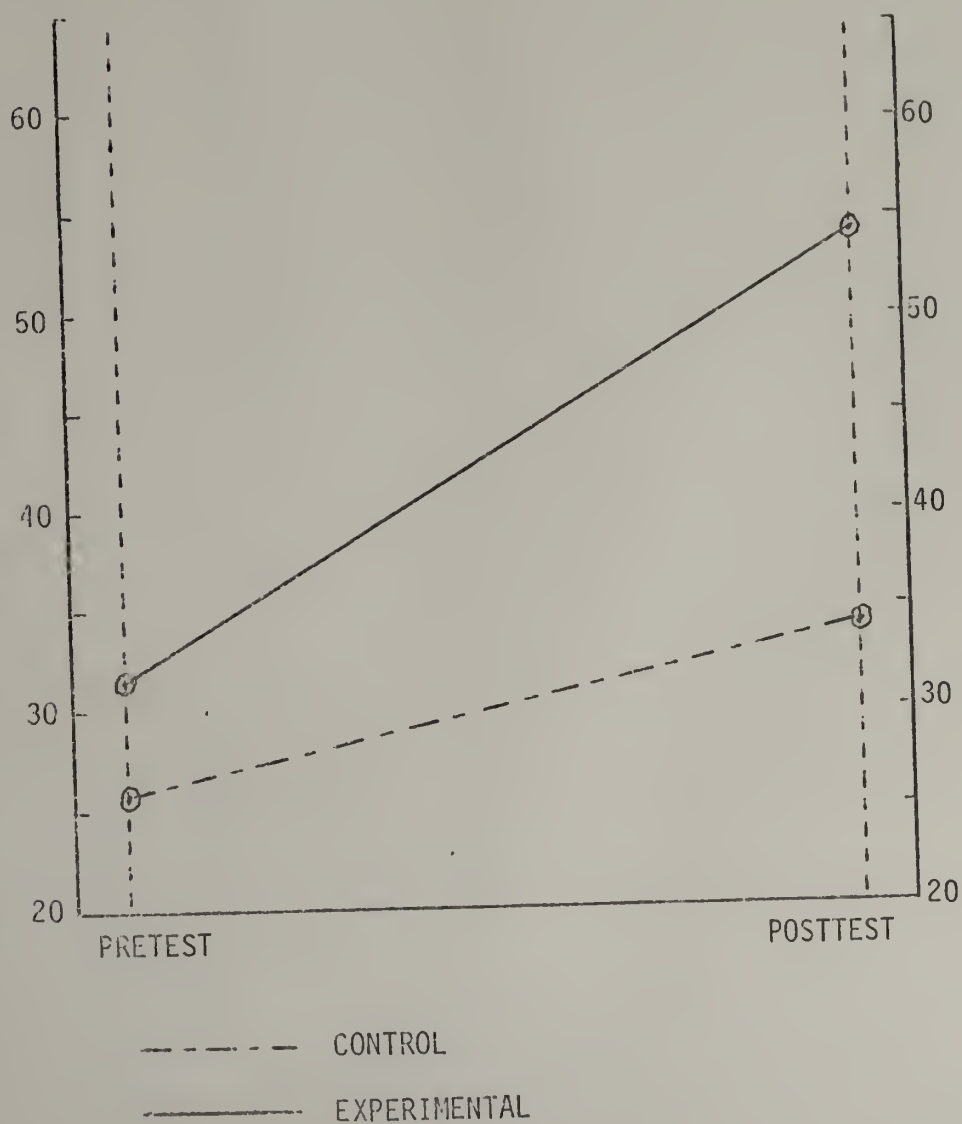
TABLE 4  
SUMMARY STATISTICS FOR WILCOXEN RANK-SUM TEST

Sum of Ranks $W$	Expected Sum of Ranks $E(W)$	Variance $\text{Var}(W)$	t-Statistic $\frac{W - E(W)}{\sqrt{\sigma_W}}$
902	689	2985.67	3.90

The observed sum of the ranks,  $W$ , for the experimental group was computed to be  $W = 902$ , while the expected sum of the ranks (assuming the distributions for the experimental and control group are alike) was 689. Using the normal approximation, this value of  $W$  corresponds to a  $t$ -statistic of 3.90. Hence, the probability of observing such a rank sum under the assumption that the control and experimental groups are independent random samples from the same distribution is less than .00005. Thus, the observed sum of ranks is certainly significant at the .05 level and the hypothesis that the computer-related modeling materials have no effect can be rejected. Exposure to the modeling curriculum materials did improve the scores of the experimental subjects as evidence by the fact that the gains made by the experimental group are significantly greater than those made by the control group. Figure 3 graphically depicts the gains made by the experimental and control groups.

The significance of the statistical results reported here is discussed in Chapter V.

FIGURE 3  
MODELING ACHIEVEMENT PRETEST AND POSTTEST MEANS  
FOR EXPERIMENTAL AND CONTROL GROUPS



## CHAPTER V

### DISCUSSION

#### Summary

The major objectives of this study were threefold: (1) the development of computer-related curriculum materials which teach mathematical modeling to seventh grade students, (2) an examination of the effectiveness of these materials as a pedagogical tool for acquiring intellectual abilities and skills related to modeling, and (3) an investigation of the relation between abstract thinking and modeling ability.

The specific objectives of the study were to test the following three null hypotheses:

1. The correlation between the student's ability to think abstractly and his/her mathematical modeling ability will be zero.
2. The mean of the difference scores on the Raven Progressive Matrices posttests for the experimental and control group will be less than or equal to zero.
3. There will be no significant difference between the distributions of the pretest-posttest difference

scores of modeling achievement for the experimental and control groups.

### Hypothesis 1

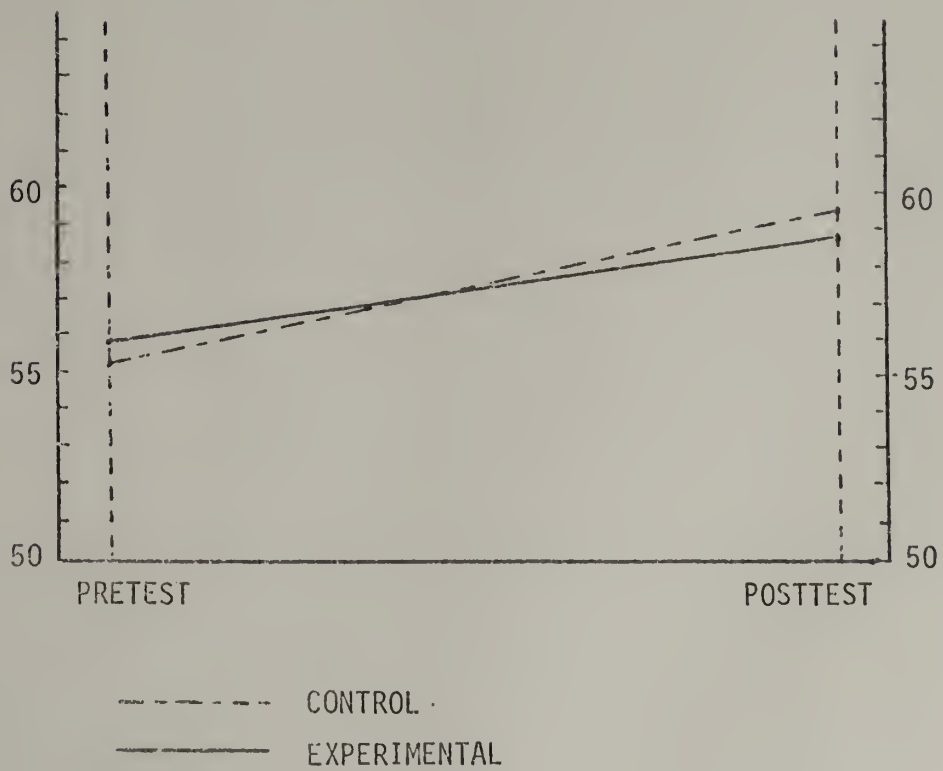
In testing Hypothesis 1, a correlation coefficient of .63 was computed between the Raven Progressive Matrices pretest scores and the modeling achievement pretest scores for the 52 students in the sample. A t-test using this correlation coefficient led to rejection of the hypothesis that the true correlation was zero. The relatively strong positive correlation of .63 found for the sample used in this study indicates that high (low) scores in the level of abstract thinking (as measured by the Progressive Matrices test) are usually found in concert with high (low) scores in the ability to construct mathematical models.

### Hypothesis 2

The mean of the difference scores on the Raven Progressive Matrices posttests for the experimental and control groups was not significantly different from zero. Hence, Hypothesis 2 could not be rejected since the statistical analysis indicated that the student's level of abstract thinking did not increase as a result of exposure to the modeling curriculum materials.

Figure 4 graphically depicts the pretest and posttest means of the experimental and control groups. Although both groups show slight gains in abstract thinking, statistical analysis of these gains indicates that they can be attributed to chance. That is, one-tailed

FIGURE 4  
RAVEN PROGRESSIVE MATRICES PRETEST AND POSTTEST MEANS  
FOR EXPERIMENTAL AND CONTROL GROUPS





t-tests run on the pretest and posttest means for each group were not significant at the .05 level. Hence, there was no significant increase in the level of abstract thinking between groups when the experimental and control group were compared, and also no significant increase in level of abstract thinking within groups.

### Hypothesis 3

In testing Hypothesis 3, it was concluded that a significant difference did exist between the distributions of the pretest-posttest difference scores of modeling achievement for the two groups. The experimental group showed significantly greater gains in modeling achievement than the control group. Hence, Hypothesis 3 was rejected and it was concluded that the curriculum materials were effective in teaching modeling to the students involved in this study.

### Conclusions

Results of this study indicated that (1) there is a positive correlation between the student's ability to reason abstractly and his/her ability to construct mathematical models, (2) the curriculum materials did not have a positive effect on the student's ability to reason abstractly, and (3) the computer-related curriculum materials developed and evaluated in this study were generally effective in teaching modeling skills and concepts to seventh grade students.

The effectiveness of the curriculum materials in teaching modeling lends support to the hypothesis that the construction of

mathematical models is a problem-solving approach that can be taught to some degree to seventh grade students irrespective of their level of abstract thinking. The Piaget-Process Matrix developed by Wood (1974) and discussed in the review of the literature suggests that children who have not reached the stage of formal operations will not be able to construct mathematical models insofar as they are not capable of formulating and testing hypotheses. While no attempt was made to assess the Piagetian developmental level of the students involved in this study, it is believed that many of them were not formal operational. This judgement is based on their ages and performance on the abstract reasoning test. The effectiveness of the curriculum materials with these students can perhaps be attributed to the fact that practice was given in formulating and testing only two types of hypotheses (the existence of linear and exponential growth patterns). Instead of emphasizing creative hypothesis building, the materials emphasized induction or the process of generalization and the ability to generalize is possessed by children at the concrete operational stage.

The failure of the curriculum materials to effect any measurable positive change in the students' level of abstract thinking may be due to the relatively short time span over which the study took place. Perhaps prolonged exposure to the techniques of modeling starting at earlier grade levels is necessary to produce a change in abstract reasoning ability. Further, the limitation of available measuring instruments and knowledge concerning the nature of abstract reasoning,

which exists in educational research, was a disadvantage to this study. A review of currently available educational and psychological testing materials reveals a lack of reliable instruments which purport to measure abstract reasoning and which can be administered to a group. The abstract reasoning portion of most group IQ tests usually involves verbal analogies and nonverbal problem figures. The Raven Progressive Matrices test was selected principally because it was designed as a group test of abstract thinking rather than as an intelligence test and because it did not involve verbal analogies which penalize poor readers. The Raven Progressive Matrices test makes no attempt to define and identify the components of abstract reasoning and simply yields a percentile score as an overall measure of abstract reasoning ability. Hence, the scale used to measure abstract reasoning may be too crude to detect changes that occurred and is not capable of detecting changes in specific aspects of abstract reasoning.

The significant difference found in testing Hypothesis 3 shows that the positive correlation found in testing Hypothesis 1 does not preclude the teaching of modeling skills to students with lower levels of abstract reasoning ability. In fact, several psychologists, including Piaget, hold that even if the ability to reason abstractly is not present to a significant degree in children, nevertheless, children must learn to reason abstractly by continual interaction with this process.

### Significance of the Study

This study has demonstrated that seventh grade students can learn mathematical modeling using a specific set of computer-related curriculum materials. Liao (1971) has shown that the modeling skills of high school students could be improved by providing them with modeling activities. This study generally substantiates and extends the findings of Liao in two respects. First, as regards the computer-related modeling materials, Liao utilized an analog computer whereas this study developed materials for implementation on a digital computer. Second, as regards age level, this study demonstrated that the modeling skills of junior high school students can also be improved by exposure to modeling curriculum materials. Consequently, the study has significance for junior high school teaching in that it shows the feasibility of teaching mathematical modeling at this grade level and provides classroom teachers with a set of curriculum materials that have been tested and shown to be effective in teaching modeling.

On a more theoretical level, the study has significance for educational research in that it attempts to shed some light on the relationship between abstract thinking and modeling ability and represents an initial investigation of materials designed to improve modeling ability. The study also points up the need for a reliable and valid instrument which measures abstract reasoning and which can be administered to a group.

### Limitations of the Study

This study had certain limitations:

1. The curriculum materials were tested with a small sample. Consequently, the generalizability of this study to other populations is limited.
2. The curriculum materials exposed students to only two types of mathematical models (linear and exponential growth models).
3. No attempt was made to compare the effectiveness of the computer-related modeling materials used in this study with curriculum materials which teach modeling without the aid of a computer. For example, curriculum materials which model through the use of physical simulations were not investigated.
4. The students' retention of the objectives over a substantial period of time was not tested.
5. Abstract thinking was operationally defined as performance on the Raven Progressive Matrices test. The limitation of available measuring instruments and knowledge concerning the nature of abstract reasoning was a disadvantage to this study.
6. The lack of availability of a measuring instrument to measure achievement in mathematical modeling necessitated the development and use of a test authored by the investigator and validated on a relatively small sample.



### Suggestions for Further Research

As a result of this study, the following recommendations are offered for future research:

1. The curriculum materials could be tested with larger and more diverse samples.
2. It could be tested whether the learning from the curriculum materials is retained by students over a substantial period of time.
3. The curriculum materials could be expanded to include different types of models, such as matrix models.
4. The curriculum materials could be compared for effectiveness with modeling curriculum materials which do not depend on the computer for their implementation.
5. Research could be done to develop valid and reliable measures of abstract thinking.
6. The reliability and validity of the mathematical modeling instrument used in this study could be improved using feedback from larger and more diverse samples.



## B I B L I O G R A P H Y

- Anastasi, A. Psychological Testing. 2nd ed. London: The Macmillan Company, 1968.
- Anderson, R. D. "Children's Ability to Formulate Mental Models to Explain Natural Phenomena." Journal of Research in Science Teaching 49 (1965): 326-332.
- Ausubel, D. P. "The Transition From Concrete to Abstract Cognitive Functioning: Theoretical Issues and Implications for Education." Journal of Research in Science Teaching 2 (1964): 261-266.
- Bell, M. S. "Mathematical Models and Applications as an Integral Part of a High School Algebra Class." The Mathematics Teacher 64 (1971): 293-300.
- Bloom, B. Taxonomy of Educational Objectives. New York: David McKay Company, 1956.
- Braun, L. "Digital Simulation in Education." Journal of Educational Technology Systems 1 (1972): 5-27.
- Bray, E. C. "MINNEMAST Elementary Mathematics-Science Program." The Physics Teacher 6 (1968): 201-206.
- Bruner, J. The Process of Education. Cambridge, Massachusetts: Harvard University Press, 1960.
- \_\_\_\_\_. On Knowing--Essays for the Left Hand. New York: Atheneum, 1962.
- Burke, H. R. "Raven's Progressive Matrices: A Review and Critical Evaluation." Journal of Genetic Psychology 93 (1958).
- Buros, O. K., ed. The Fourth Mental Measurements Yearbook. Highland Park, New Jersey: The Gryphon Press, 1953.
- \_\_\_\_\_, ed. The Sixth Mental Measurements Yearbook. Highland Park, New Jersey: The Gryphon Press, 1965.
- Chester, M. Graze Ecology Simulation. Cupertino, California: Hewlett-Packard Company, 1973.

Cambridge Conference on the Correlation of Science and Mathematics in the Schools. Goals for the Correlation of Elementary Science and Mathematics. Boston, Massachusetts: Houghton-Mifflin Company, 1969.

Computer Assisted Mathematics Program. CAMP. Glenview, Illinois: Scott, Foresman and Company, 1968.

Conference Board of the Mathematical Sciences, National Advisory Committee on Mathematical Education. Overview and Analysis of School Mathematics, Grades K-12. Washington, D. C., 1975.

Dorn, W. S. "Simulation Versus Models: Which One and When?" Journal of Research in Science Teaching 12 (1975): 371-377.

Dunfee, M. Elementary School Science: A Guide to Current Research. Washington, D. C.: Association for Supervision and Curriculum Development, NEA, 1967.

Dwyer, T. A. "Some Principles for the Human Use of Computers in Education." International Journal of Man-Machine Studies 3 (1971): 219-239.

Engineering Concepts Curriculum Project (ECCP). The Man-Made World. New York: McGraw-Hill Book Company, 1971.

The ESS Reader. Newton, Massachusetts: Education Development Center, Inc., 1970.

Feurzeig, W., et al. Programming-Languages as a Conceptual Framework for Teaching Mathematics. Cambridge, Massachusetts: Bolt, Beranek and Newman, Inc., 1969.

Feurzeig, W., and Lukas, G. "LOGO--A Programming Language for Teaching Mathematics." Educational Technology 12(3), (1972): 39-46.

Flavell, J. H. The Developmental Psychology of Jean Piaget. Princeton, New Jersey: D. Van Nostrand Company, Inc., 1963.

Gagne, R. M. The Conditions of Learning. 2nd ed. New York: Holt, Rinehart and Winston, Inc., 1965.

\_\_\_\_\_. "Elementary Science: A New Scheme of Instruction." Science 151 (1966): 49-53.

Ginsberg, H., and Oppen, S. Piaget's Theory of Intellectual Development. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1969.

- Griffith, J., and Morrison, P. "Reflections on a Decade of Grade-School Science." Physics Today 25(6), (1972): 29-34.
- Gronlund, N. E. Measurement and Evaluation in Teaching. 2nd ed. New York: The Macmillan Company, 1971.
- Hays, W. L. Statistics for Psychologists. New York: Holt, Rinehart and Winston, 1963.
- Jacobson, W., and Kondo, A. SCIS Elementary Science Sourcebook. Berkeley, California: The Science Curriculum Improvement Study, 1968.
- Karplus, R. "Physics for Beginners." Physics Today 25(6), (1972): 36-47.
- Karplus, R., and Thier, H. D. A New Look at Elementary School Science. Chicago: Rand McNally and Company, 1967.
- Klassen, D., and James, D. "Computers as Instructional Tools for the Social Sciences." Journal of Educational Technology 1, (1972): 113-120.
- Lehmann, E. L. Nonparametrics: Statistical Methods Based on Ranks. San Francisco: Holden-Day, Inc., 1975.
- Liao, T. "The Use of Analog Computer Simulation for Learning Modeling Concepts and Skills." Doctoral dissertation, Columbia University, 1971.
- Lien, A. J. Measurement and Evaluation of Learning. Dubuque, Iowa: Wm. C. Brown Company Publishers, 1967.
- Marshall, J. C., and Hales, L. W. Classroom Test Construction. Reading, Massachusetts: Addison-Wesley Publishing Company, 1971.
- Massachusetts Advisory Council on Education. Essentially Elementary Science. A report on the status of elementary science in Massachusetts schools, March 1973.
- Meadows, D. H., et al. The Limits to Growth. New York: Universe Books, 1972.
- Minsky, M. "Matter, Mind, and Models." In M. Minsky (ed.), Semantic Information Processing. Cambridge, Massachusetts: The MIT Press, 1968.
- \_\_\_\_\_. "Form and Content in Computer Science." Journal of the Association for Computing Machinery 17(2), (1970): 197-215.

- McIntyre, P. J. "Students' Use of Models in Their Explanations of Electrostatic Phenomena." Science Education 58 (1974): 577-580.
- Overview. Minneapolis: Minnesota Mathematics and Science Teaching Project, 1970.
- Paidy, L. G. "Physicists and the Elementary Schools--New Strategies for an Old Problem." American Journal of Physics 40 (1972): 576-583.
- Papert, S. "A Computer Laboratory for Elementary Schools." Computers and Automation 21(6), (1972): 19-23. (a)
- \_\_\_\_\_. "On Making a Theorem for a Child." Proceedings of the ACM Annual Conference (August 1972). (b)
- \_\_\_\_\_. "Teaching Children Thinking." Mathematics Teaching 58 (1972): 2-7. (c)
- \_\_\_\_\_. "Teaching Children to be Mathematicians Versus Teaching About Mathematics." International Journal of Mathematics, Education, Science and Technology 3 (1972): 249-262. (d)
- Papert, S., and Solomon, C. "Twenty Things to Do With a Computer." Educational Technology 12(4), (1972): 9-18.
- Pasquino, A., and Peelle, H. A. "Teaching Ecology With a Programming Language." The American Biology Teacher 37 (1975): 488-491.
- Peckham, H. D. Air Pollution. Cupertino, California: Hewlett-Packard Company, 1973.
- Piaget, J. "Physical World of the Child." Physics Today 25(6), (1972): 23-27.
- Raven, R. J. "Programming Piaget's Logical Operations for Science Inquiry and Concept Attainment." Journal of Research in Science Teaching 11 (1974): 251-261.
- Saaty, T. L. Topics in Behavioral Mathematics. Washington, D. C.: Mathematical Association of America, 1973.
- Science--A Process Approach. AAAS Miscellaneous Publication 67-12. Maryland: AAAS Commission on Science Education, 1968.
- SCIS Sample Guide. Berkeley, California: The Science Curriculum Improvement Study, 1970.
- Suchman, J. R. "The Illinois Studies in Inquiry Training." Journal of Research in Science Teaching 2 (1964): 230-232.



- Thorndike, R. L., ed. Educational Measurement. 2nd ed.  
Washington, D. C.: American Council on Education, 1971.
- Williams, D. L., and Herman, W. L. Current Research in Elementary School Science. New York: The Macmillan Company, 1971.
- Wood, D. A. "The Piaget-Process Matrix." School Science and Mathematics 74 (1974): 407-412.
- Ziegler, R. E. "The Relative Effectiveness of the Use of Static and Dynamic Models in Teaching Elementary School Children the Theoretical Concept--The Particle Nature of Matter." Doctoral dissertation, University of Wisconsin, 1967.

APPENDIX A  
TAXONOMY OF MODELS



## TAXONOMY OF MODELS

## I. Conceptual (Abstract) Models

The mental or conceptual representation of some real physical object or phenomena. These representations have their existence in the mind but such models may be made concrete by means of mathematical expressions, computer programs, simulations, games, etc.

## A. Nonmathematical Models

Analogies with no quantification, plausible explanations of phenomena or scientific theories.

Examples: Freud's theory, DeBono's models of the functions of the brain system, Minsky's "Frames."

## B. Mathematical Models

## 1. Quantitative

Models based on the complex number system.  
Models in which measurement is involved.

## a. Continuum

Models involving the interval or ratio scales of measurement. This kind of model is common in the physical sciences.

Example: Models in the physical sciences in which real and complex numbers are used.

## b. Discrete

Models which involve a mild form of measurement, i.e., the nominal and ordinal scales. Models in the social sciences generally fall into this category. Included also

based on counting, models involving probabilities, models which use graph theory and matrix theory models from finite mathematics.

## 2. Qualitative

Models primarily based on logic and the algebra of sets, but which are not reducible to numbers. No measurement is involved in qualitative models. Most models in the social sciences and some physical science models fall into this category. Models in this category use axiomatics, set theory, group theory and certain aspects of graph theory.

Examples: Piaget's sixteen binary operations and INRC group model.  
T. Saaty's graph theory model of the Instant  
Insanity colored cubes.

## II. Physical (Concrete) Models

The physical representation of some real physical object or phenomena. Since physical models must exist in the mind before they exist physically, all physical models stem from mental models.

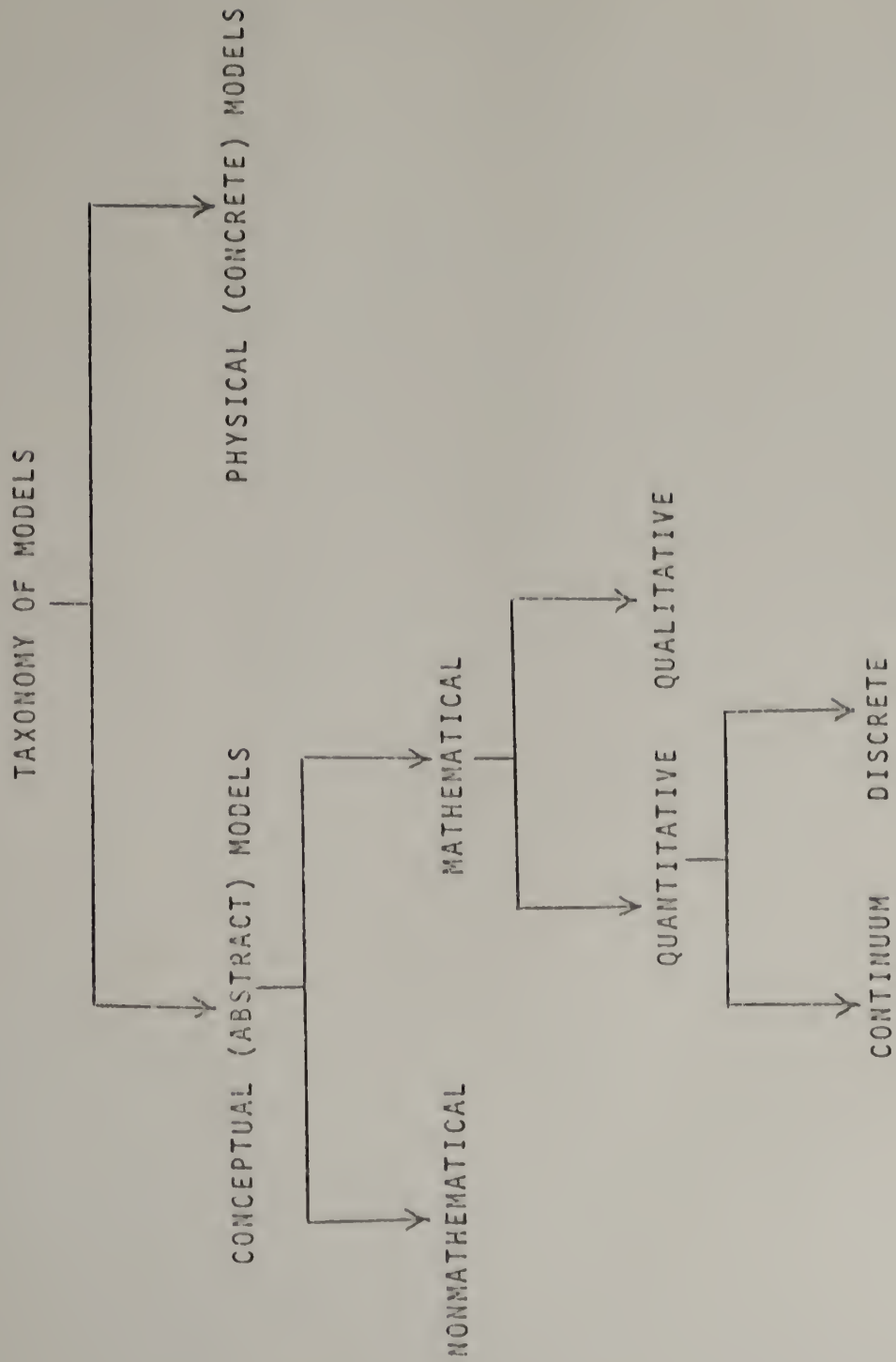
Physical models may take one of the following forms or combination of forms:

Scaled down replica--e.g., a model airplane, model railroad, tropical fish tank, erector set constructions.

Computer program--e.g., programs used in the Limits to Growth model of the world.

Simulations--e.g., ping-pong balls in an air stream model Brownian motion. Also simulated environments such as a wind tunnel, Link trainer, planetarium, zero-gravity chamber for astronauts.

Games--e.g., Monopoly models business transactions. The game of Life models cell growth and the Sumerian Game models a civilization and planning for its survival.



## APPENDIX B

## RAVEN PROGRESSIVE MATRICES

Standard  
Progressive Matrices

Sets A, B, C, D and E

Prepared by J. C. PAVEN, M.Sc.  
Printed in Great Britain  
at the University Printing House, Cambridge  
Published by H. K. Lewis & Co., Ltd. London

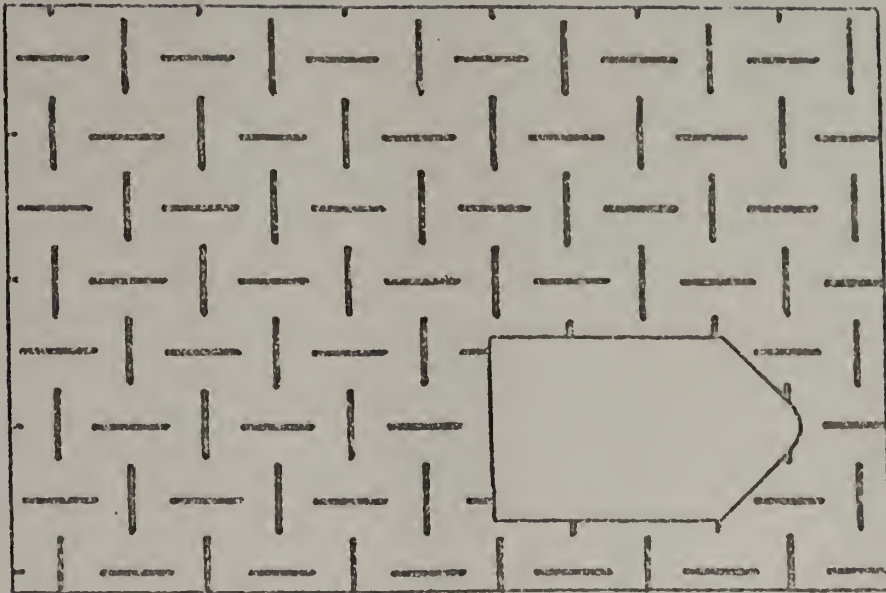
JOHN C. PAVEN  
1935

<i>Printed for Experimental Work</i>	1936
<i>Revised and Standardized Series published</i>	1938
<i>Reprinted with extended Norms</i>	1940
<i>Reprinted</i>	1941
"	1942
"	1943
"	1944
"	1945
"	1946
"	1947
<i>Revised and published with Norms for people 6 to 65 years of age</i>	1948
<i>Reprinted</i>	1951
<i>Reprinted with Revised Order of Problems and Wrong Choices</i>	1956
<i>Reprinted</i>	1958
"	1961
"	1964
"	1965
"	1969
"	1972
"	1974



## SET A

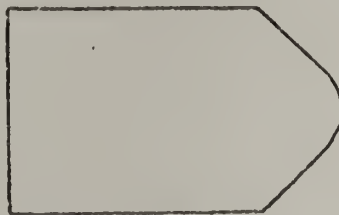
A 1



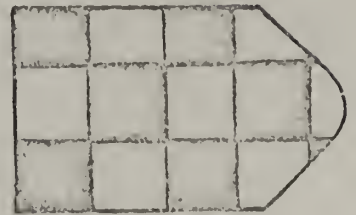
1



2



3



4



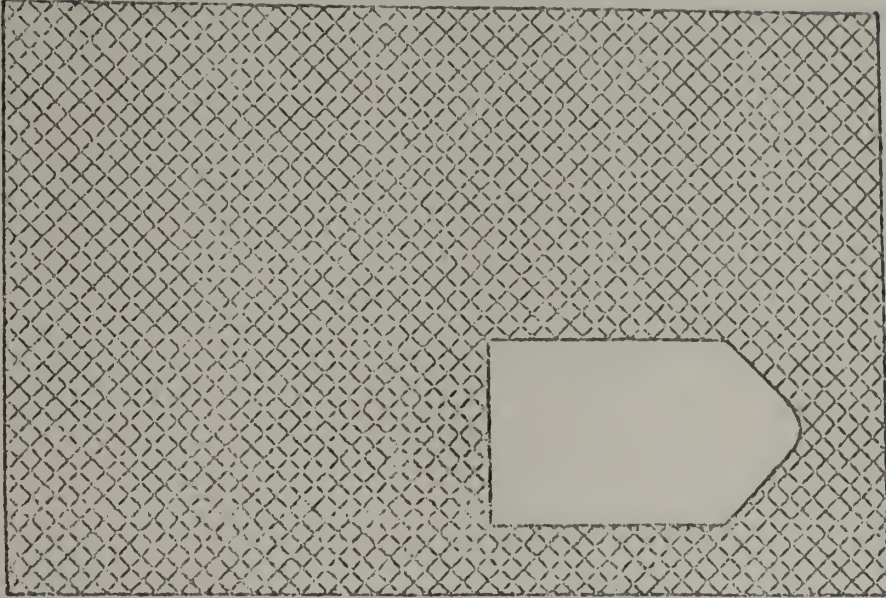
5



6



A 2



1



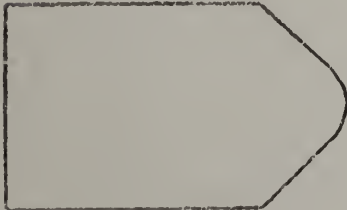
2



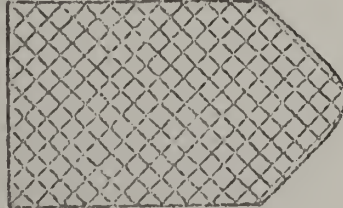
3



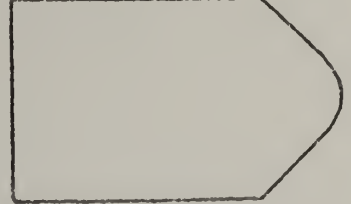
4



5



6

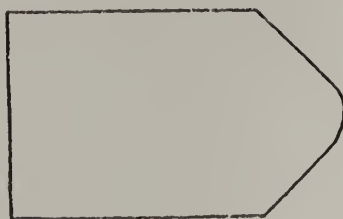


A<sub>3</sub>

1



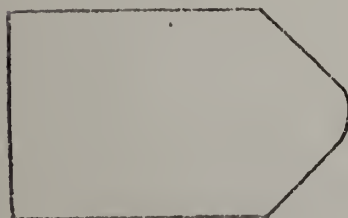
2



3



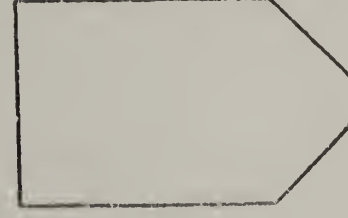
4



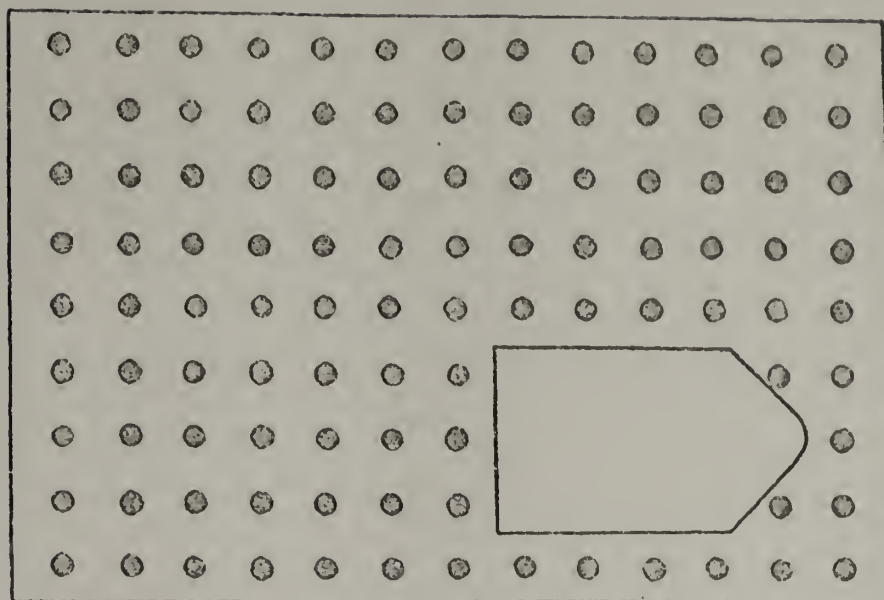
5



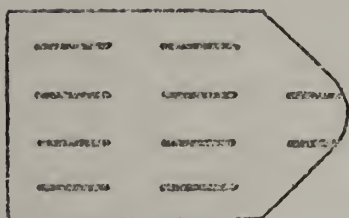
6



A 4



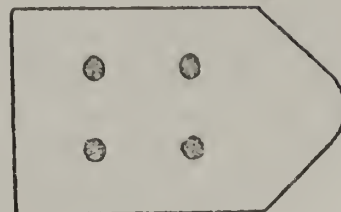
1



2



3



4



5

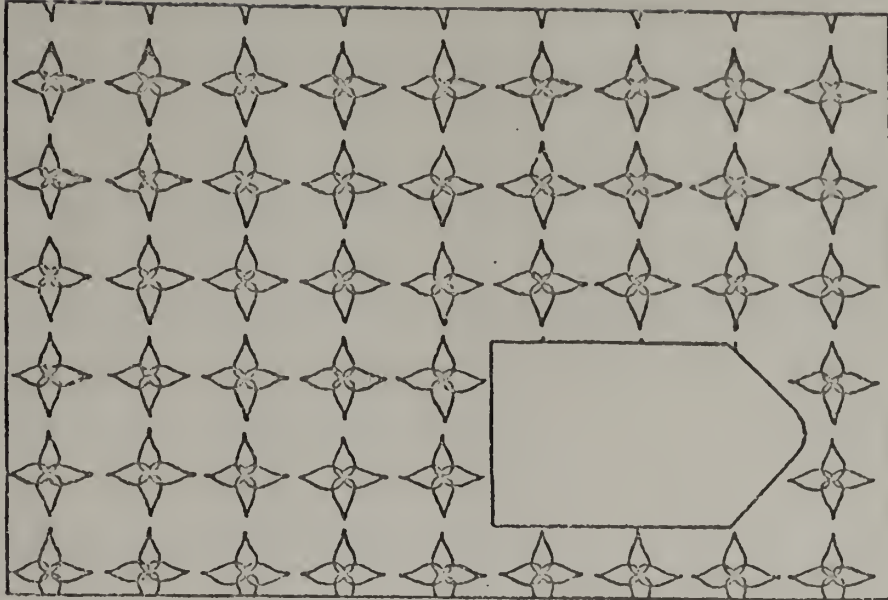


6





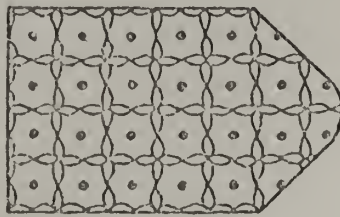
A 5



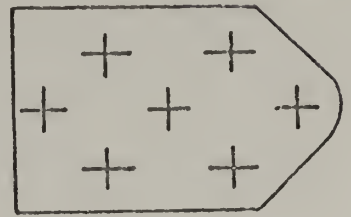
1



2



3



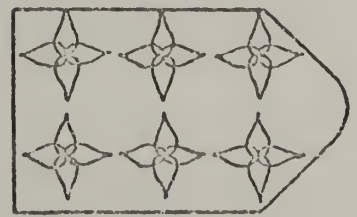
4



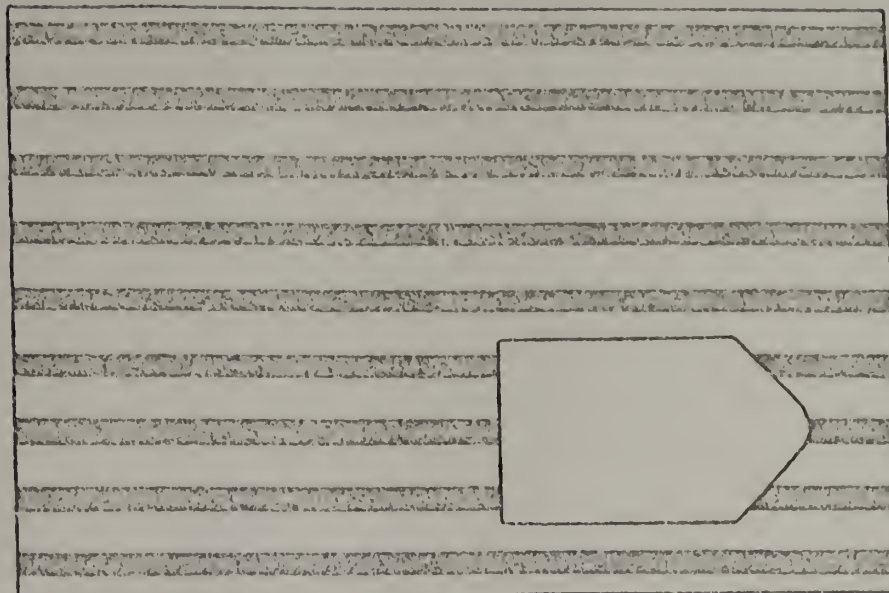
5



6



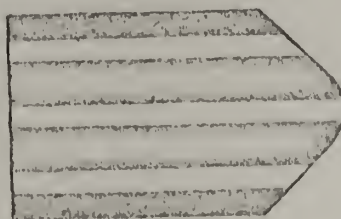
A 6



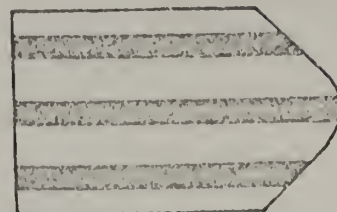
1



2



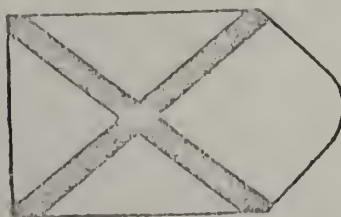
3



4



5

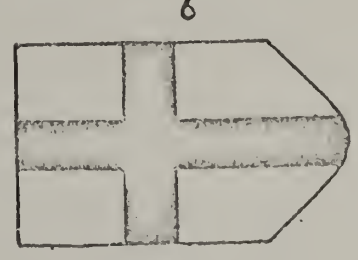
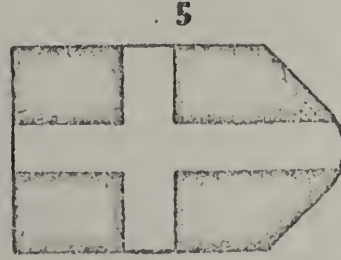
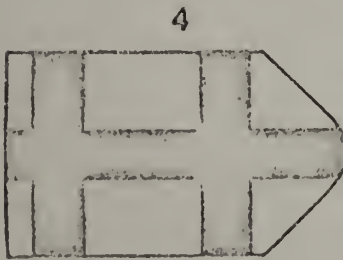
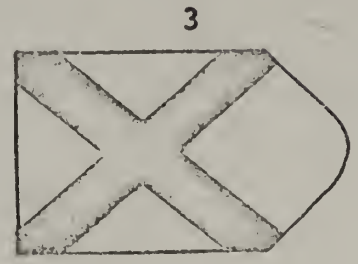
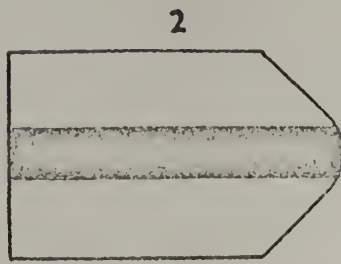
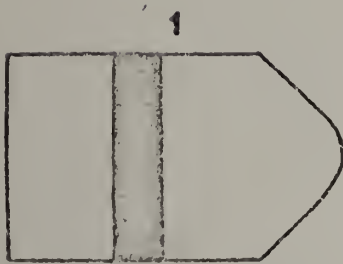
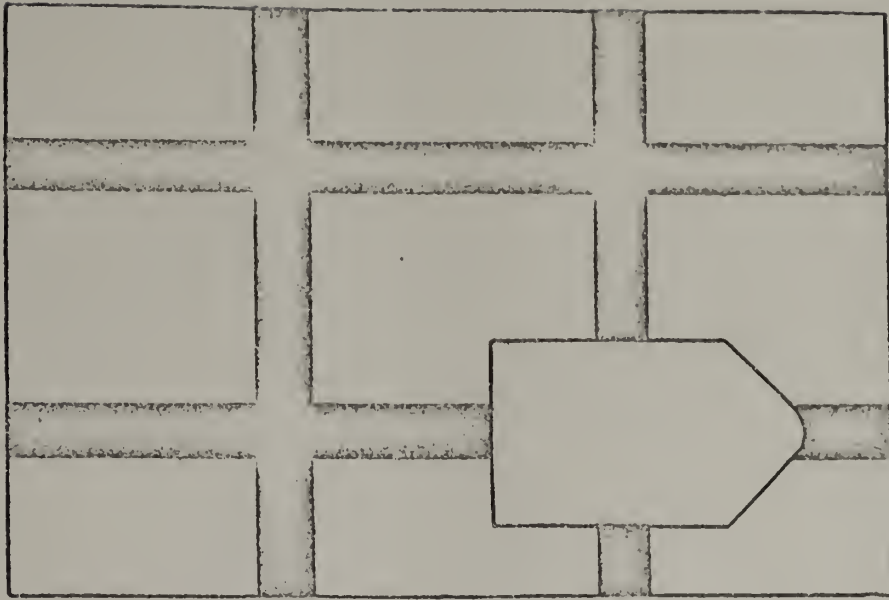


6

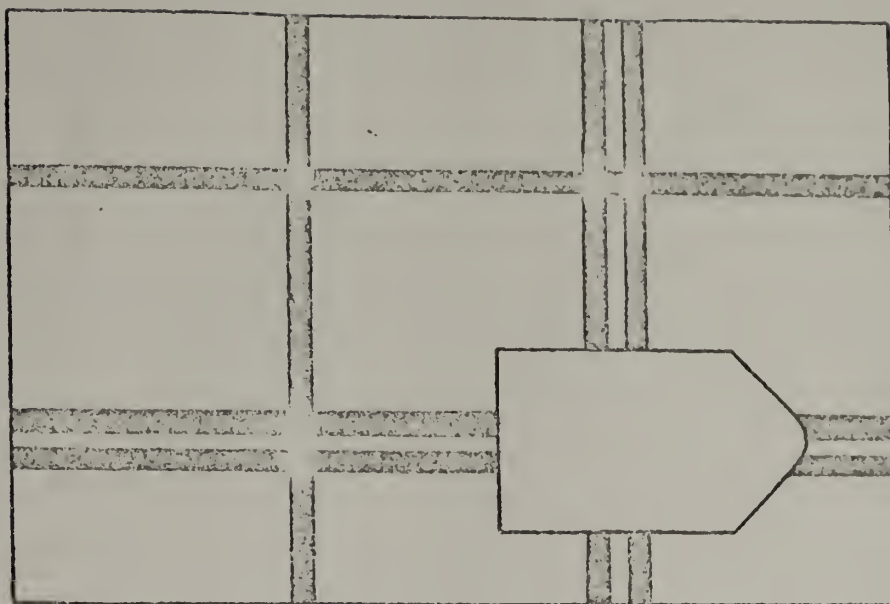




A7



A 8



1



2



3



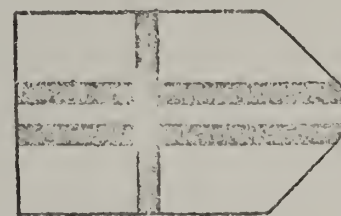
4



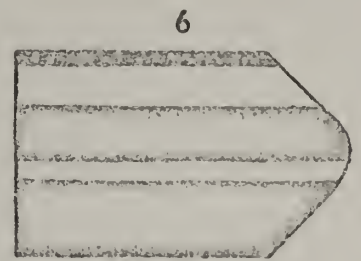
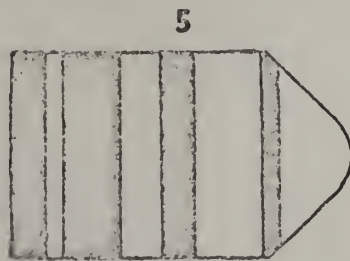
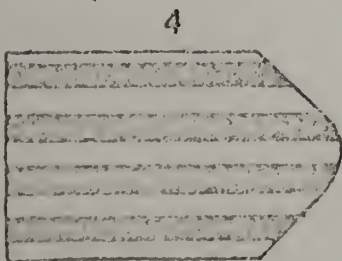
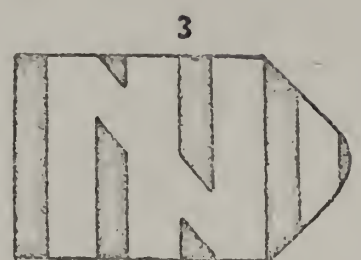
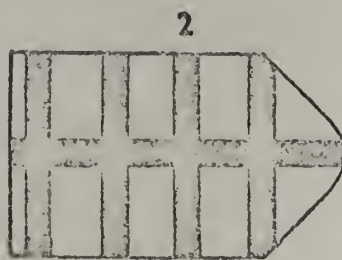
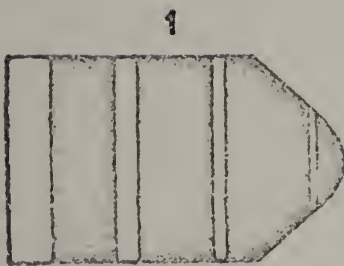
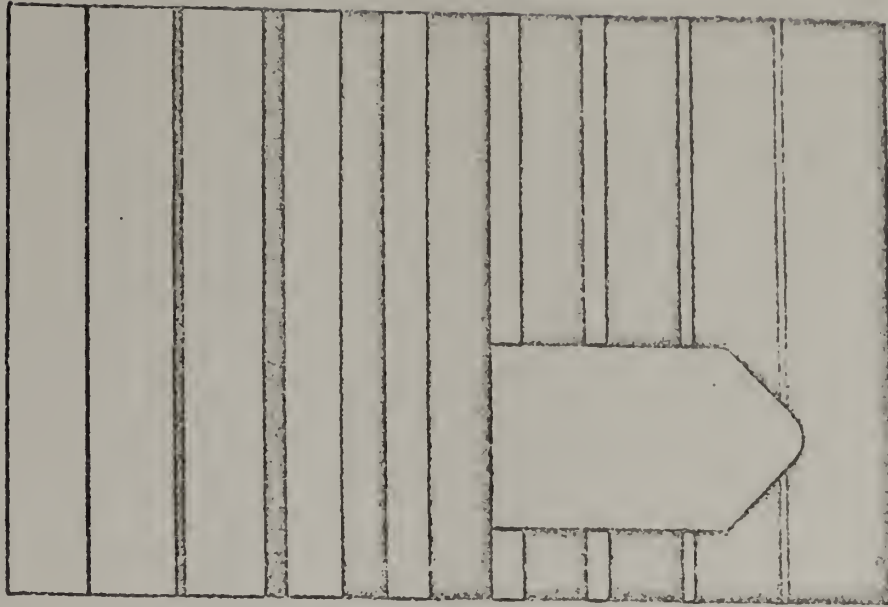
5



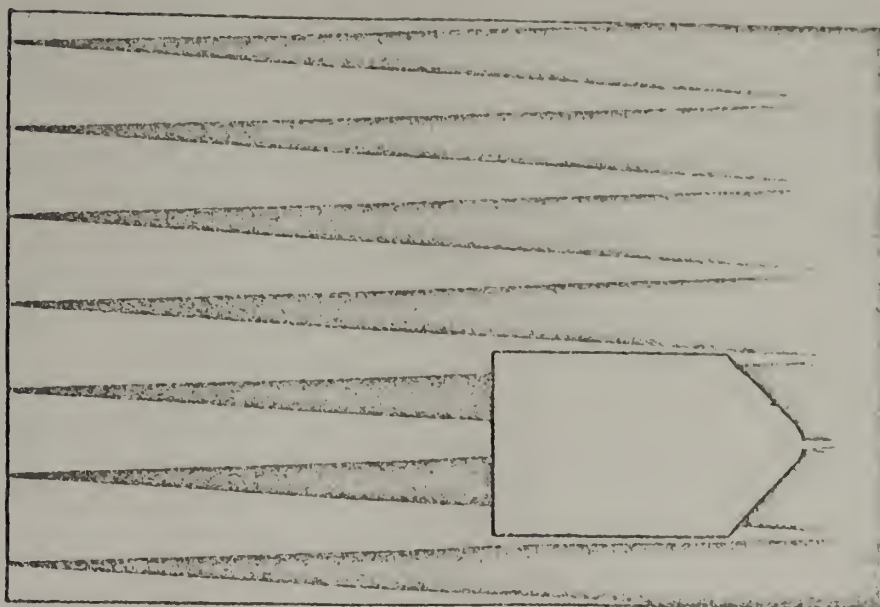
6



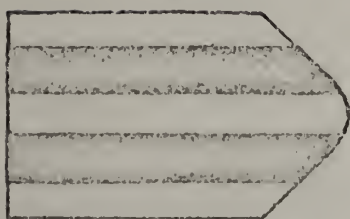
A 9



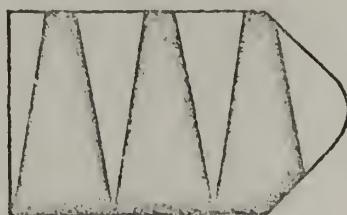
A 10



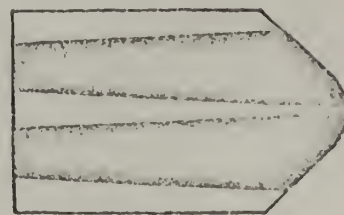
1



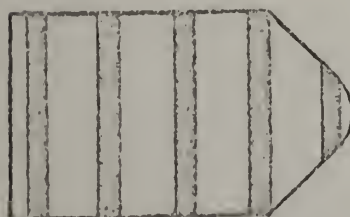
2



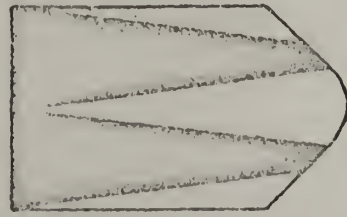
3



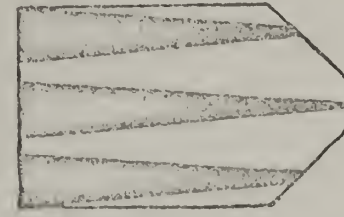
4



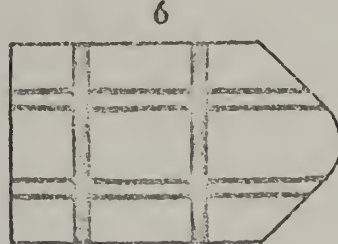
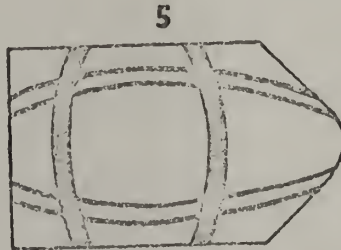
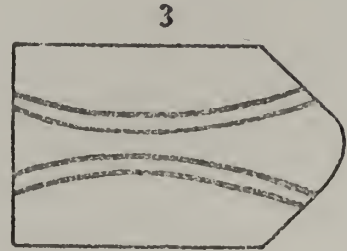
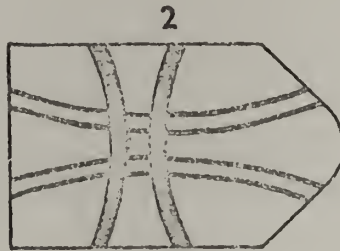
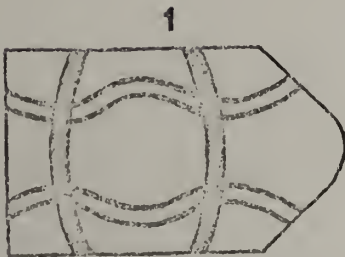
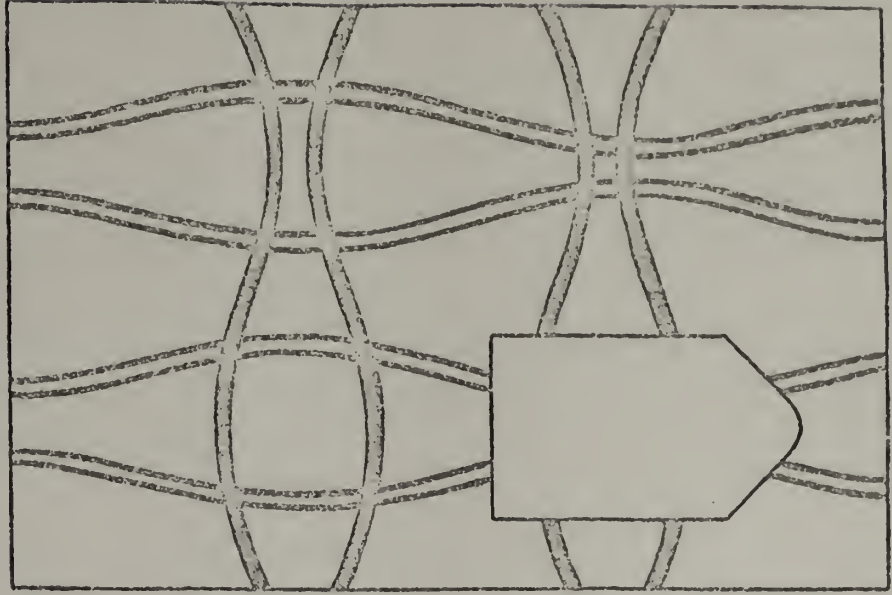
5



6

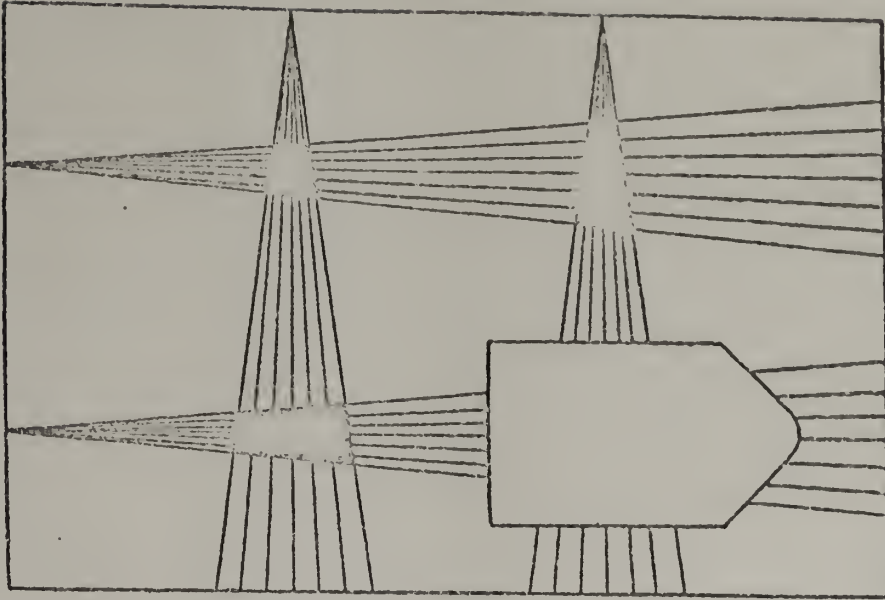


## A II

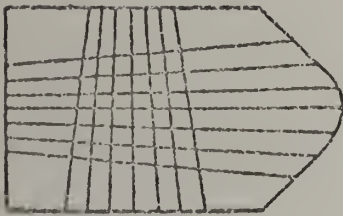




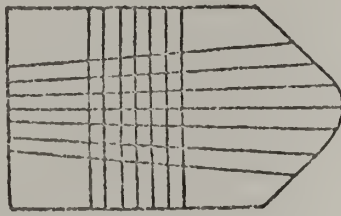
A 12



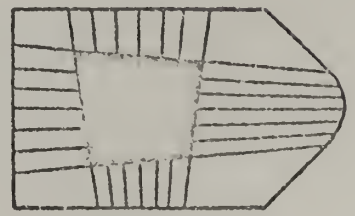
1



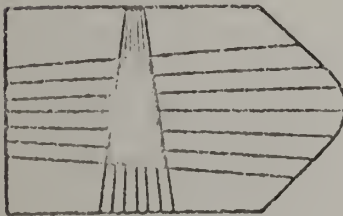
2



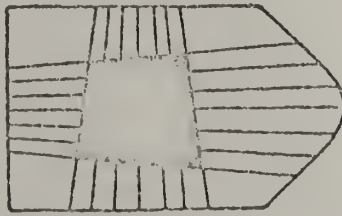
3



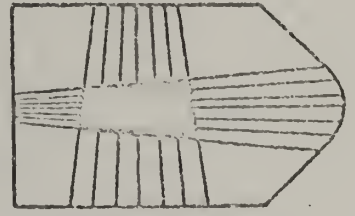
4



5



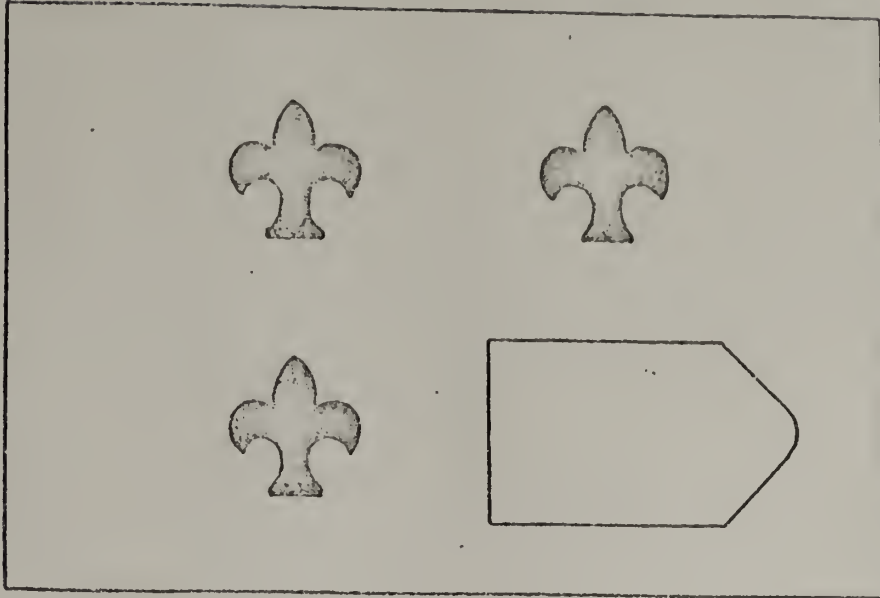
6





## SET B

B 1



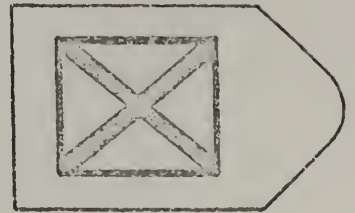
1



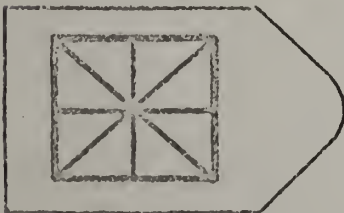
2



3



4



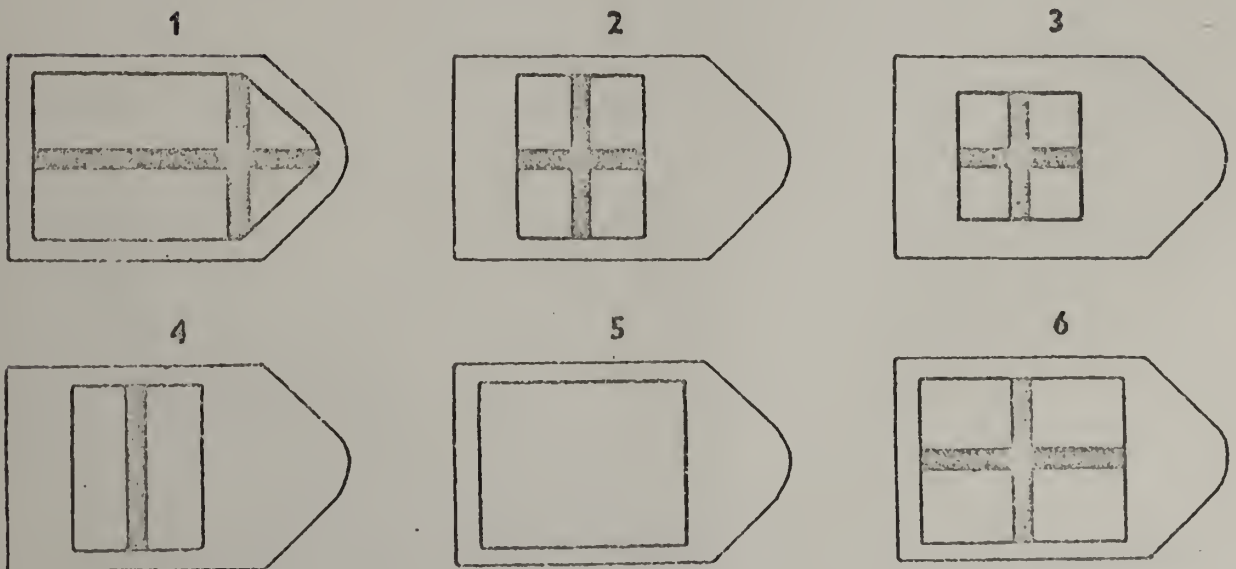
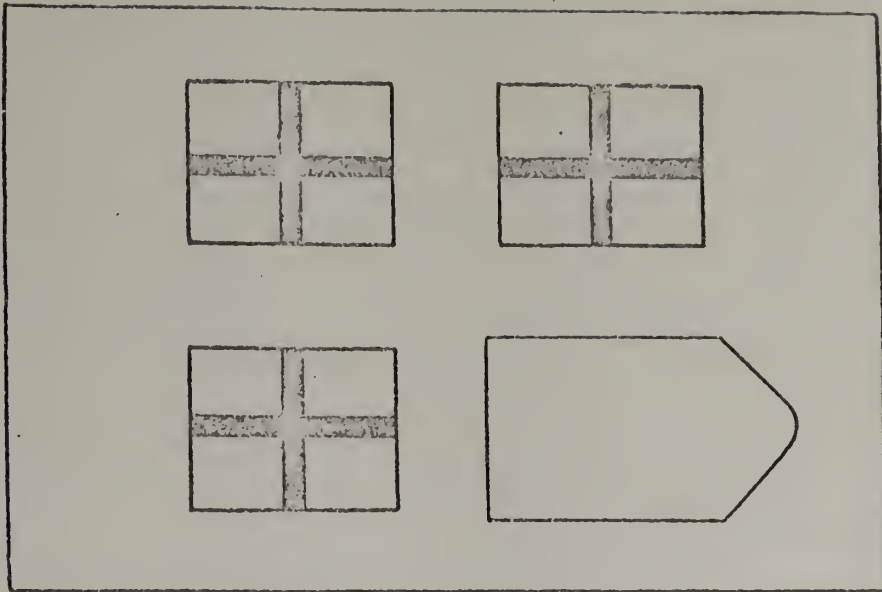
5

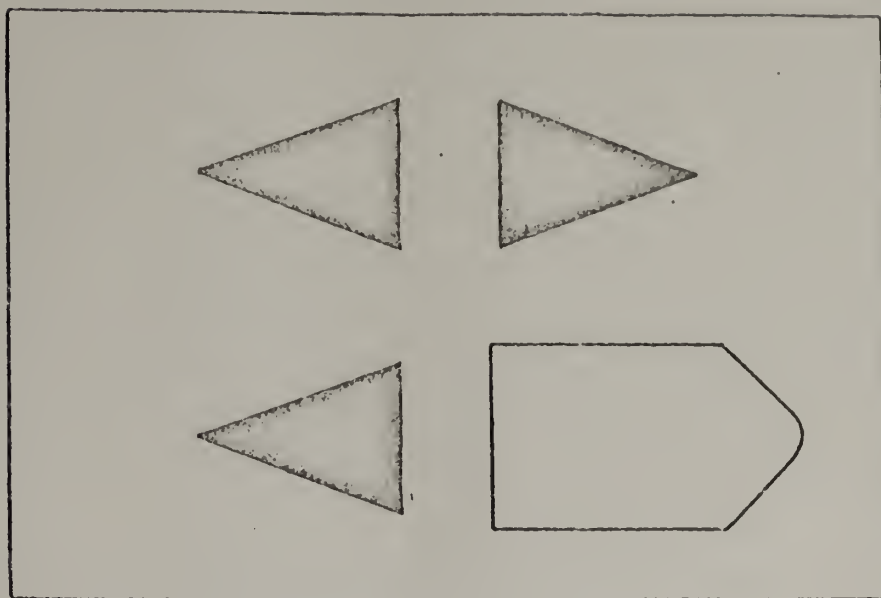


6



B 2



B<sub>3</sub>

1



2



3



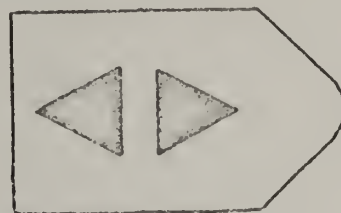
4

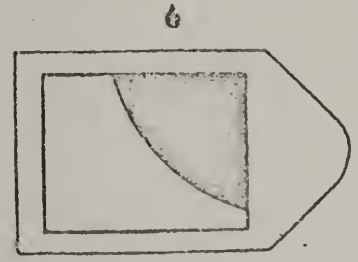
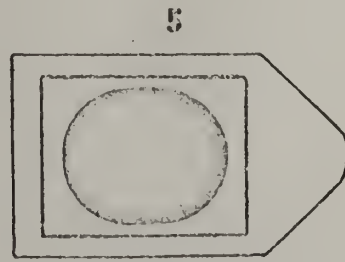
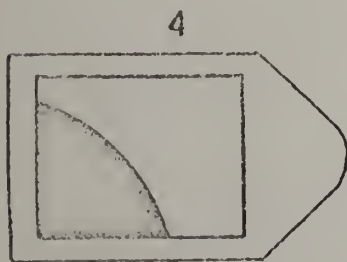
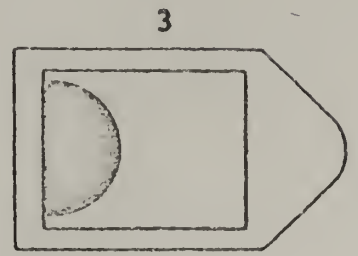
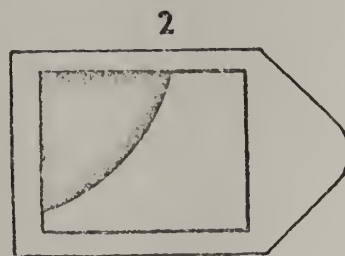
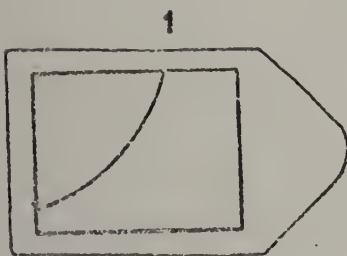
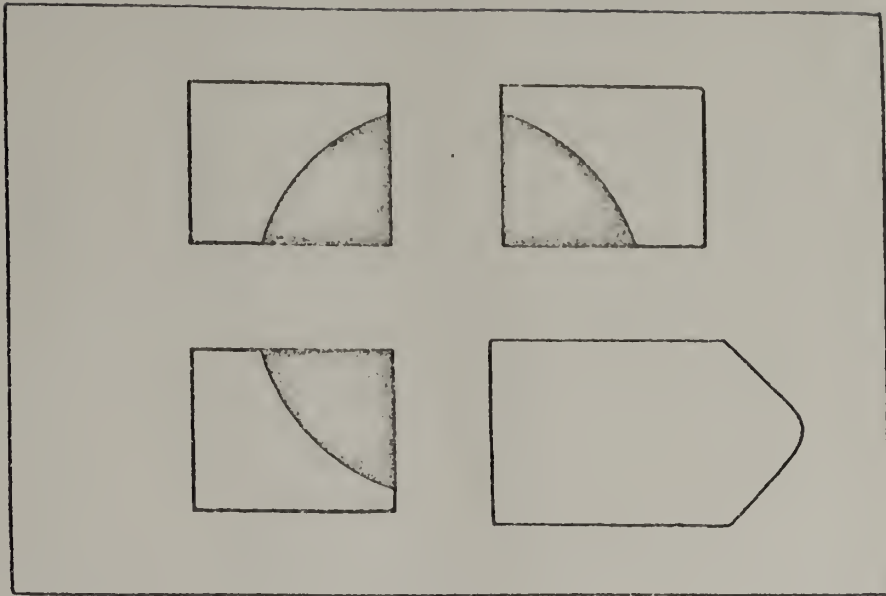


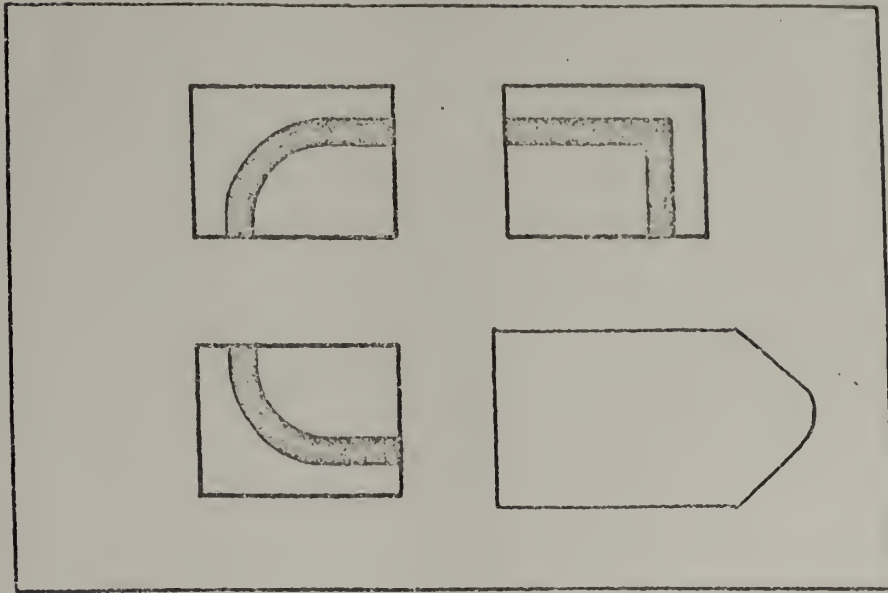
5



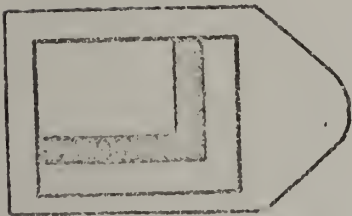
6



B<sub>4</sub>

B<sub>5</sub>

1



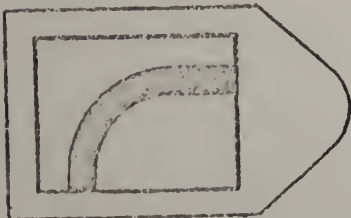
2



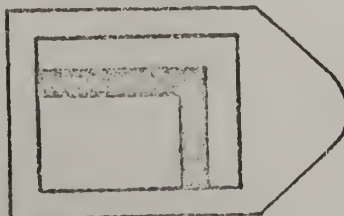
3



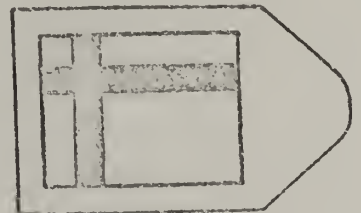
4



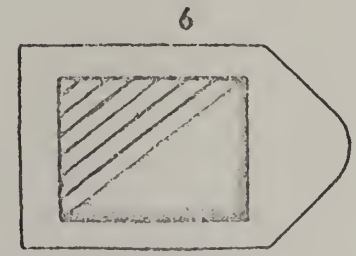
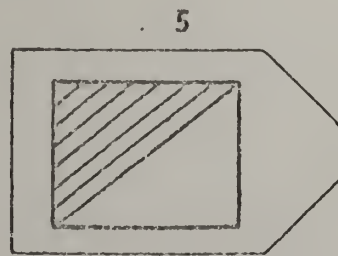
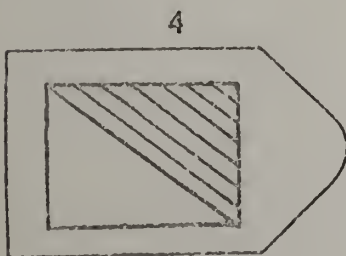
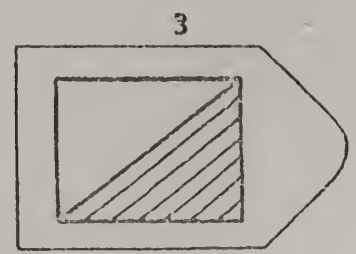
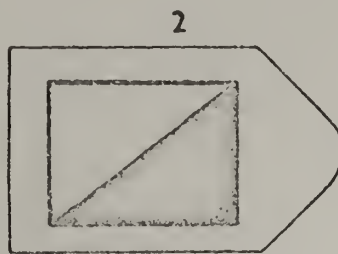
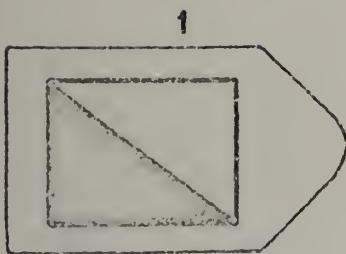
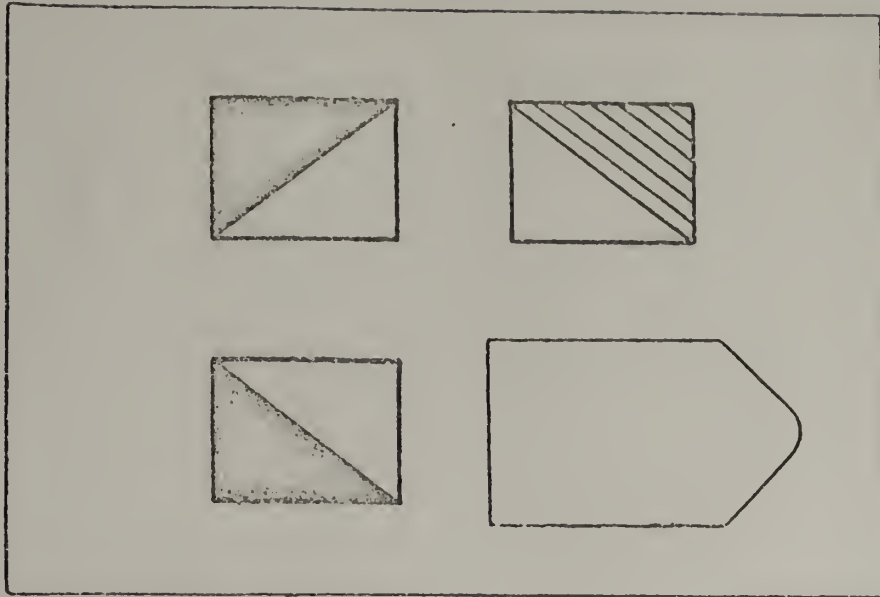
5



6

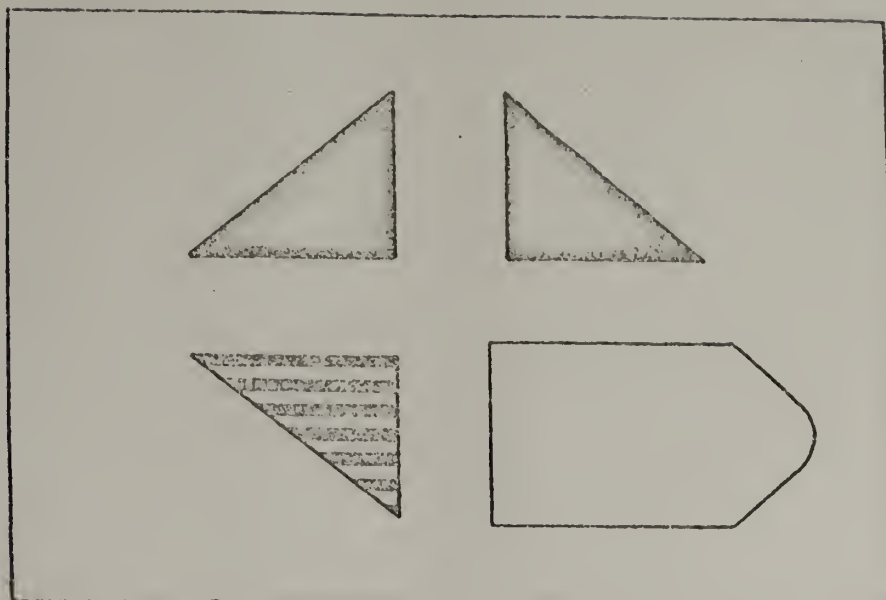


B 6

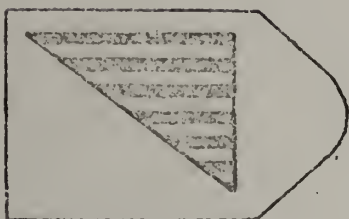




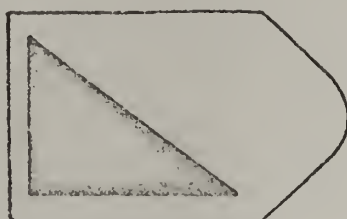
B 7



1



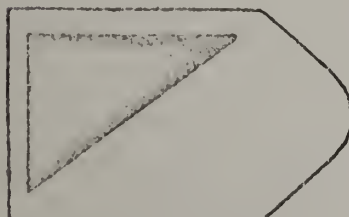
2



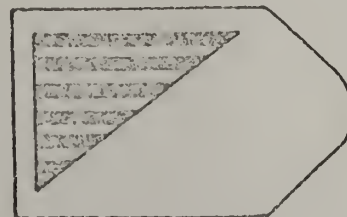
3



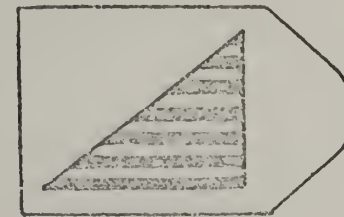
4



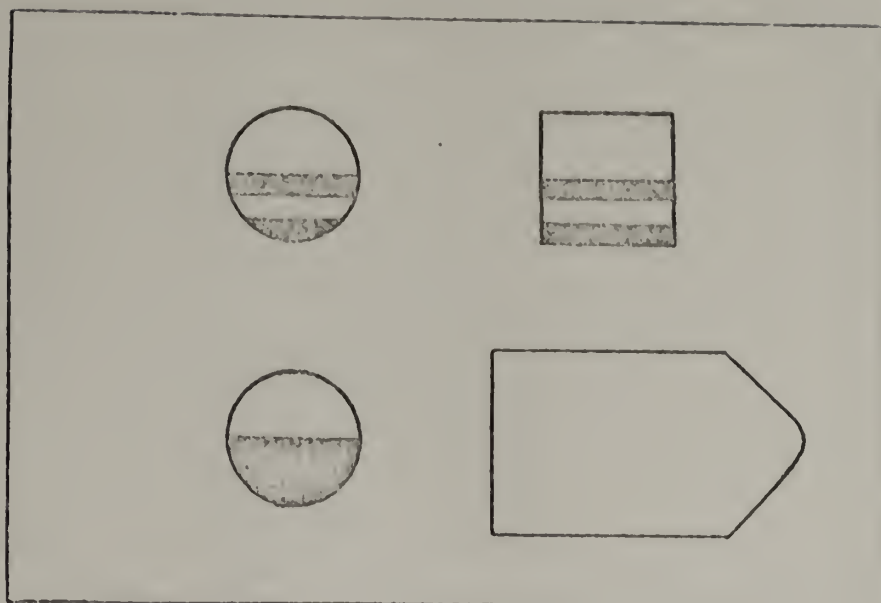
5



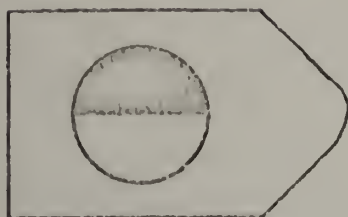
6



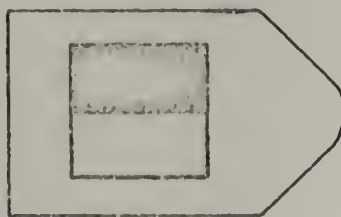
B 8



1



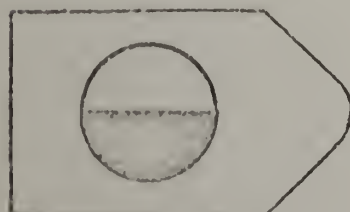
2



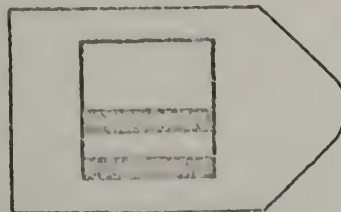
3



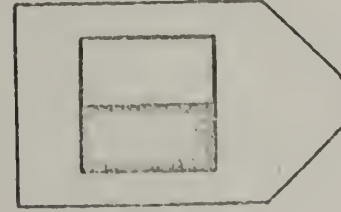
4



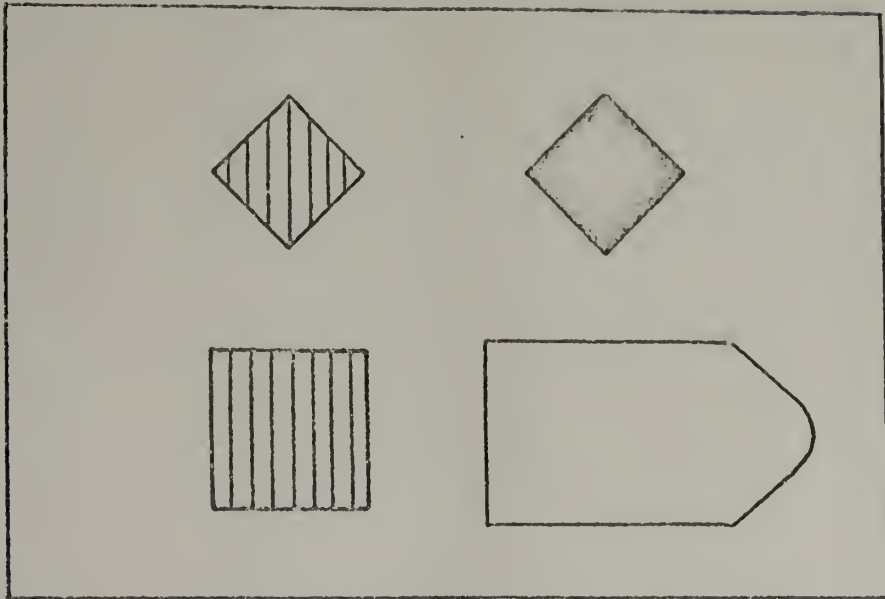
5



6



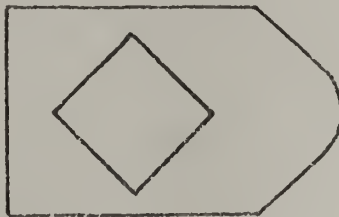
B9



1



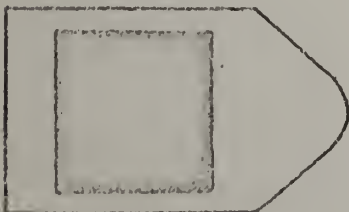
2



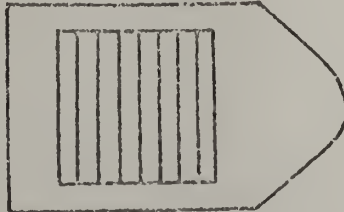
3



4



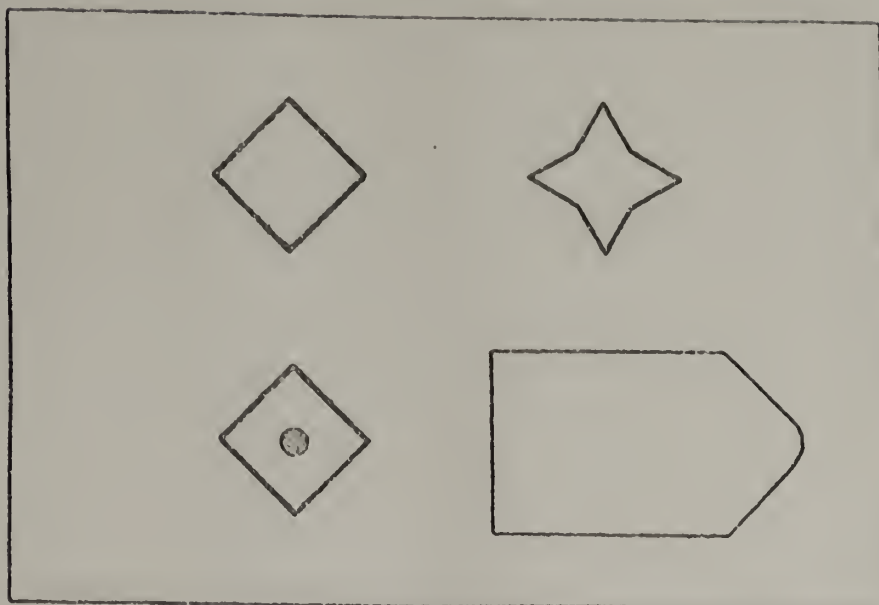
5



6



B 10



1



2



3



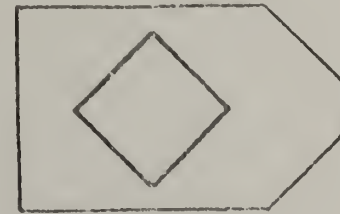
4



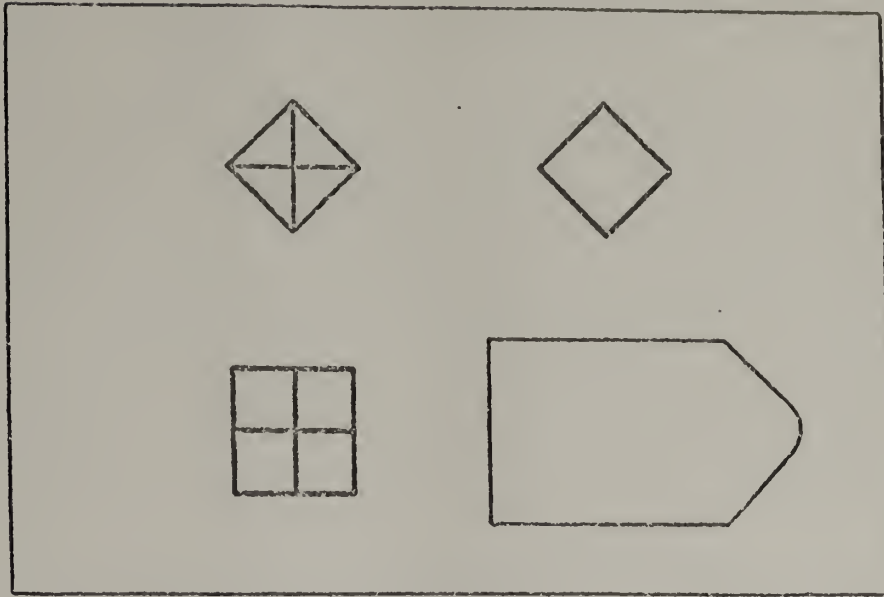
5



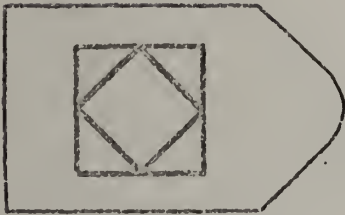
6



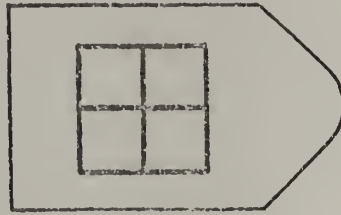
## B II



1



2



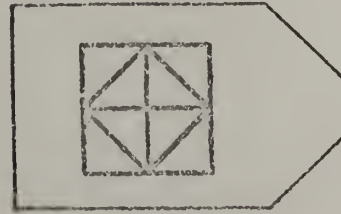
3



4



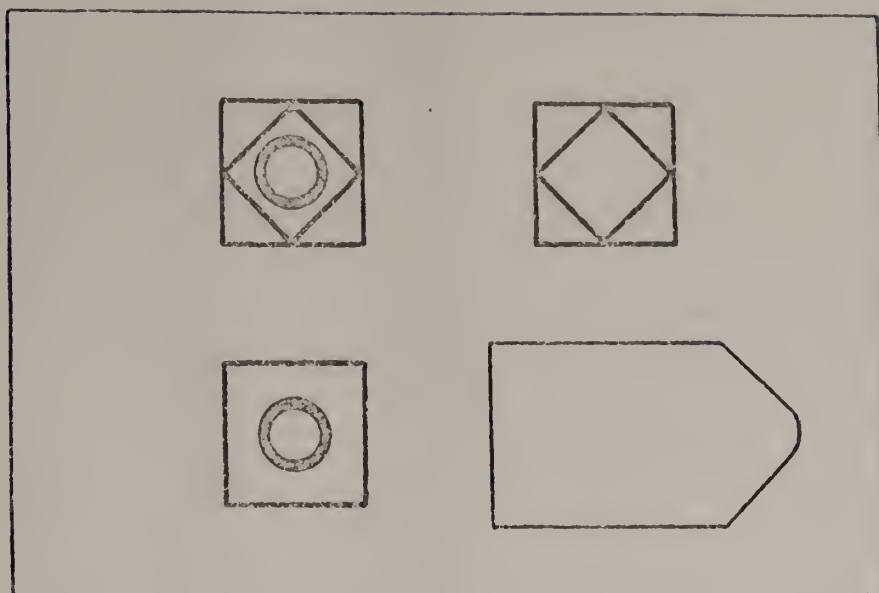
5



6



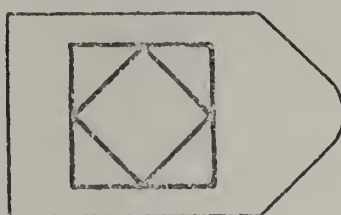
B 12



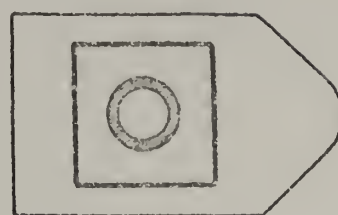
1



2



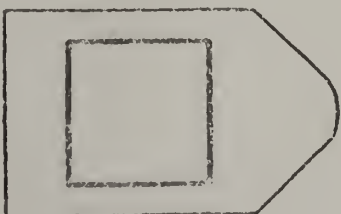
3



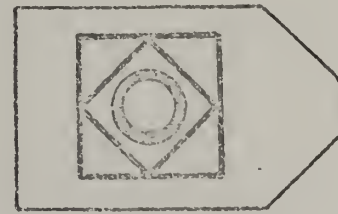
4



5

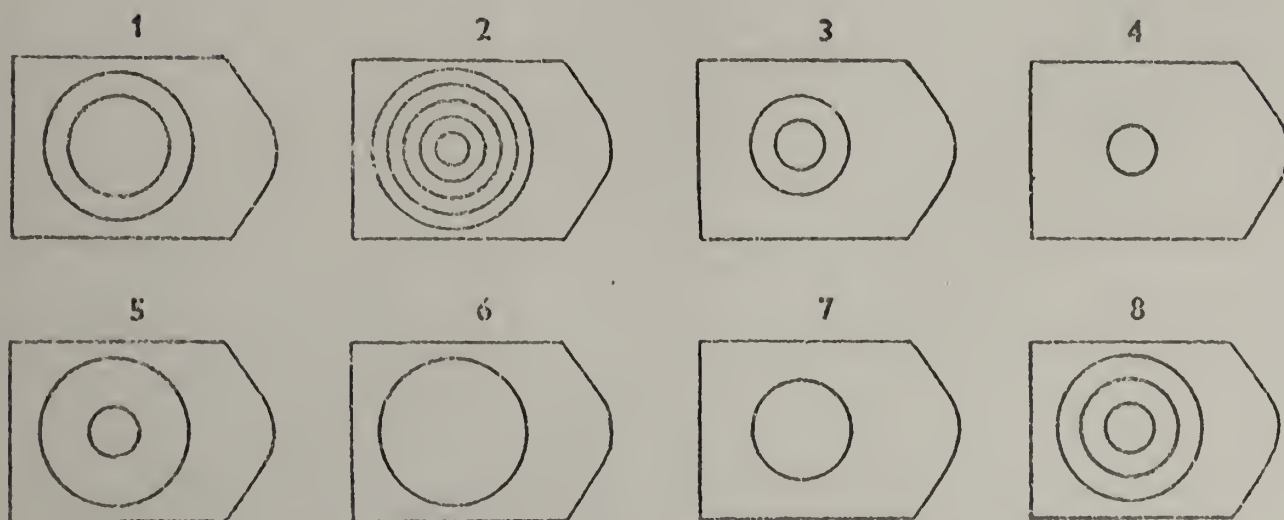
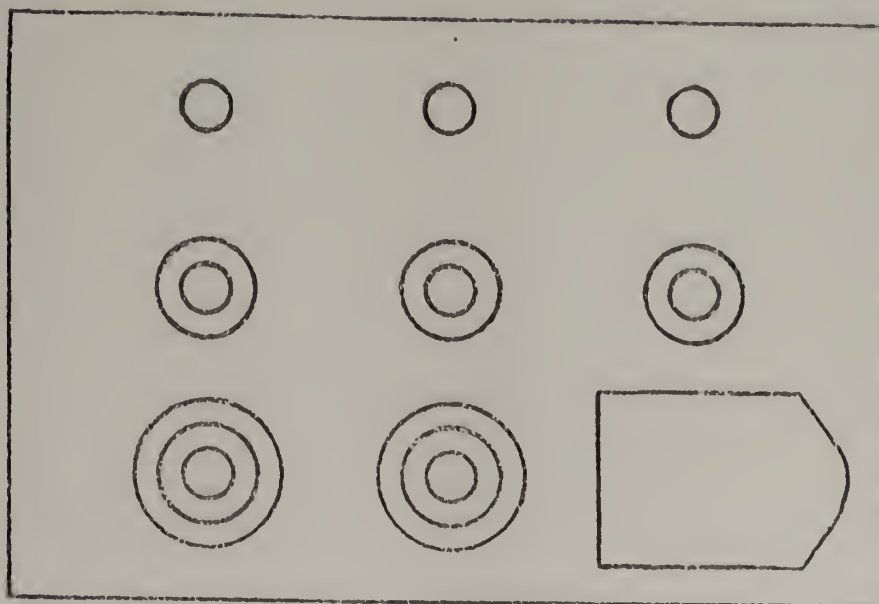


6

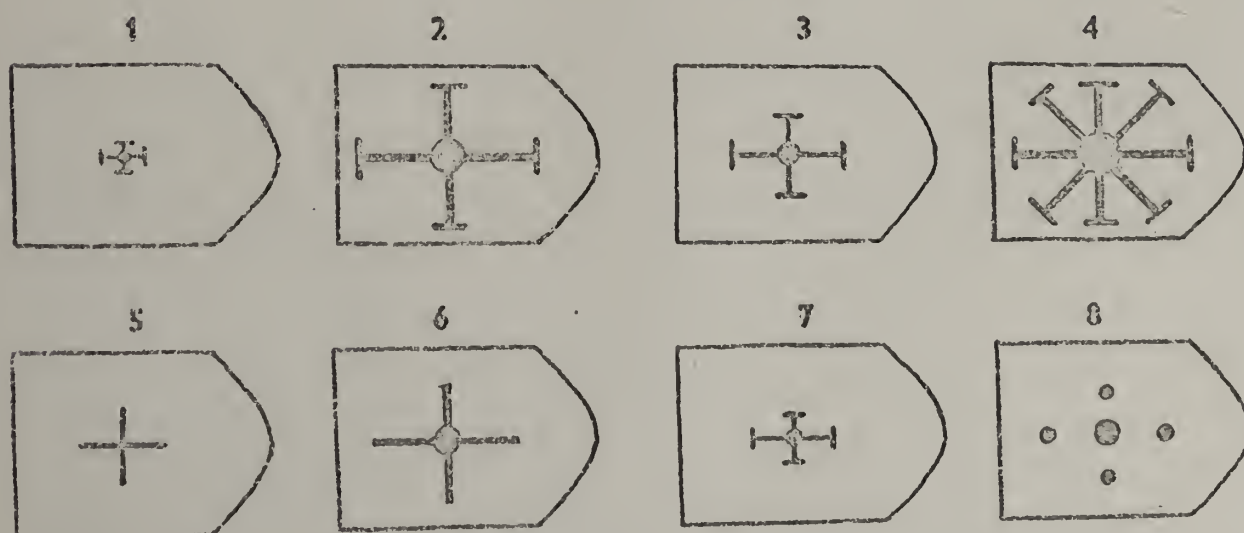
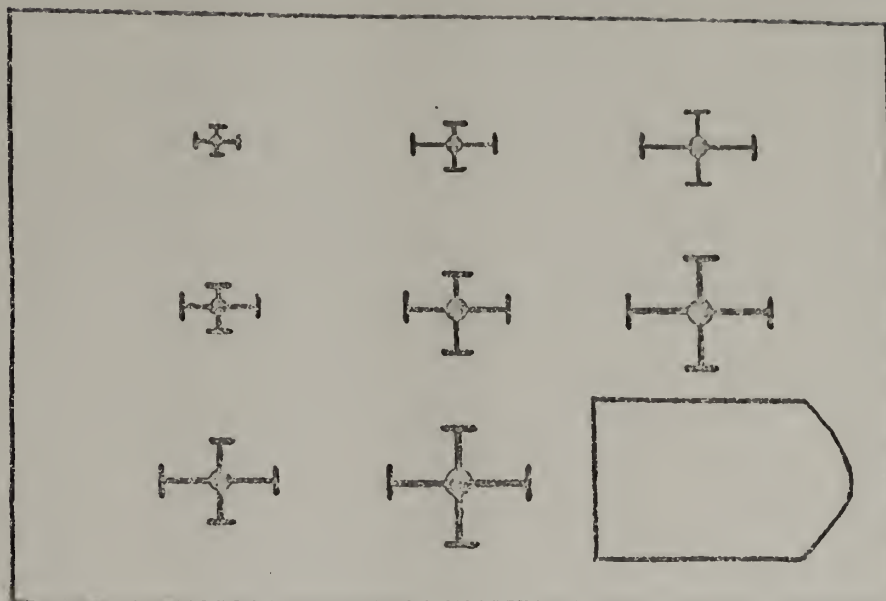


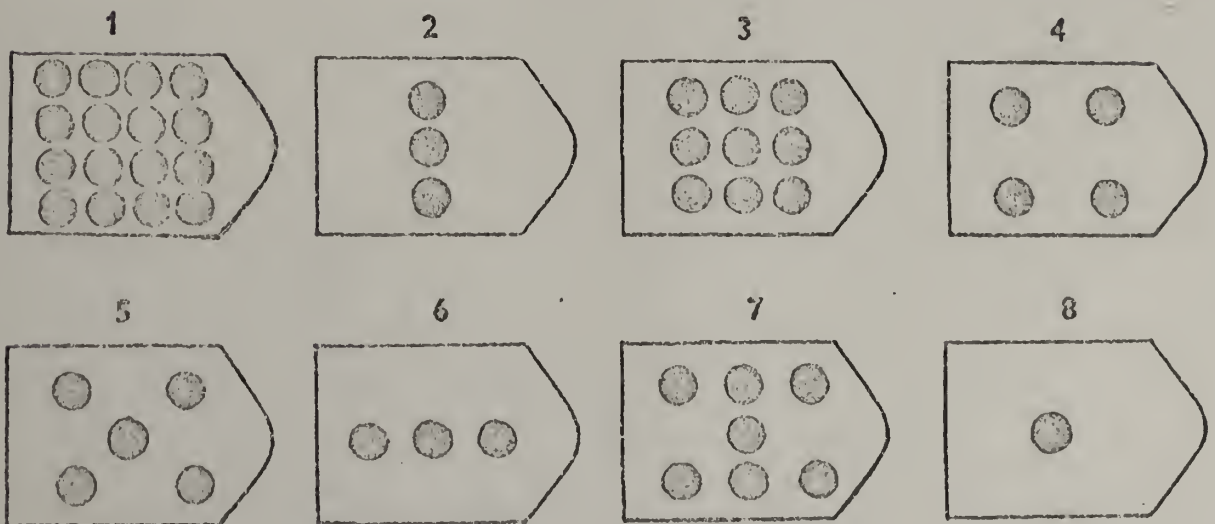
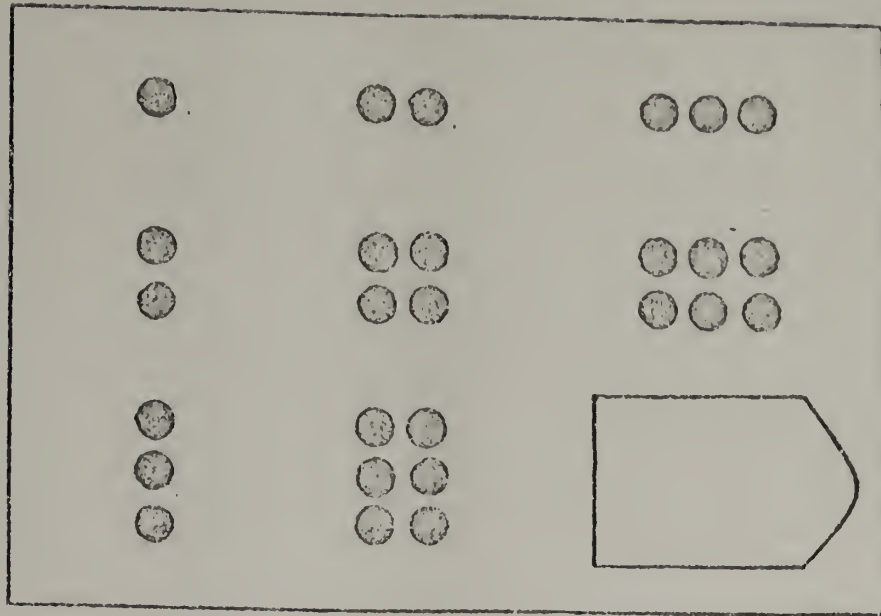


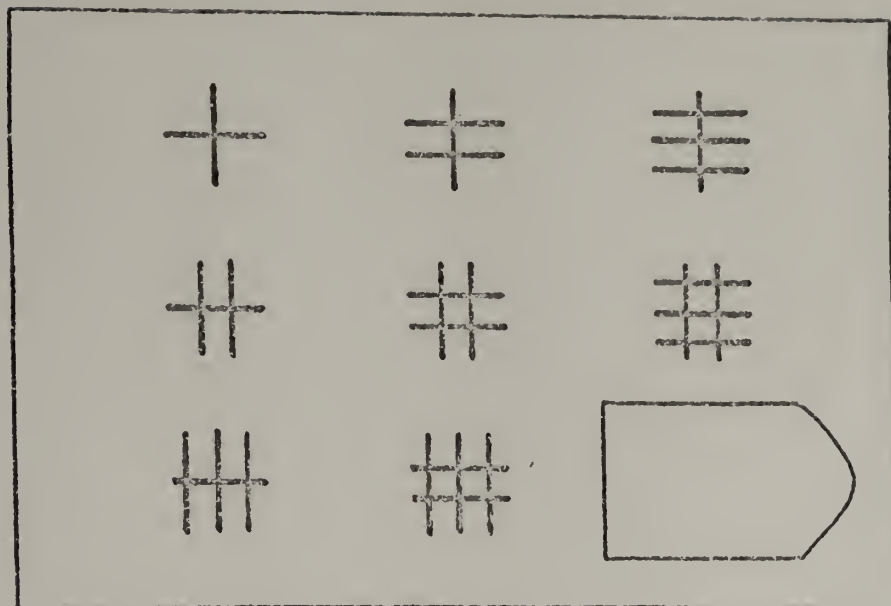
## SET C

C<sub>I</sub>

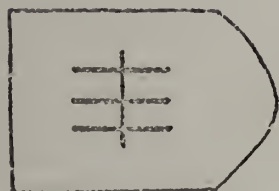
C 2



C<sub>3</sub>

C<sub>4</sub>

1



2



3



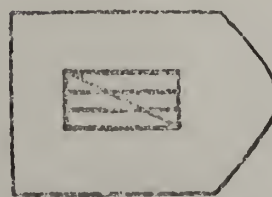
4



5



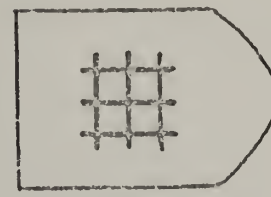
6



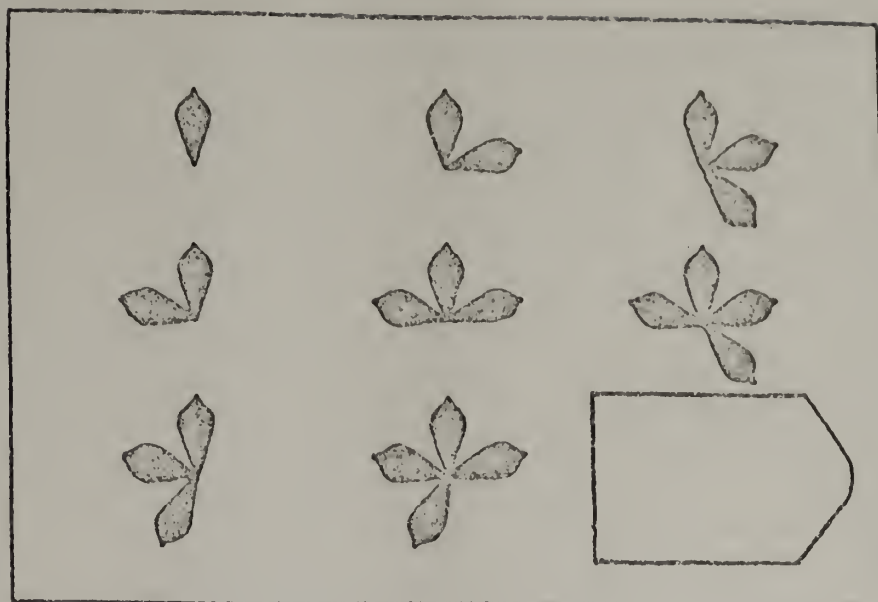
7



8



C5



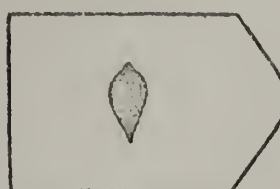
1



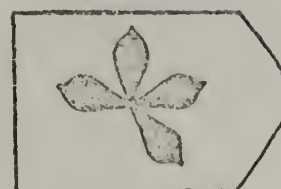
2



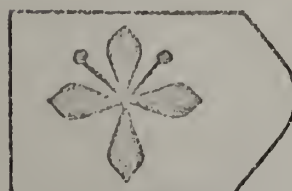
3



4



5



6



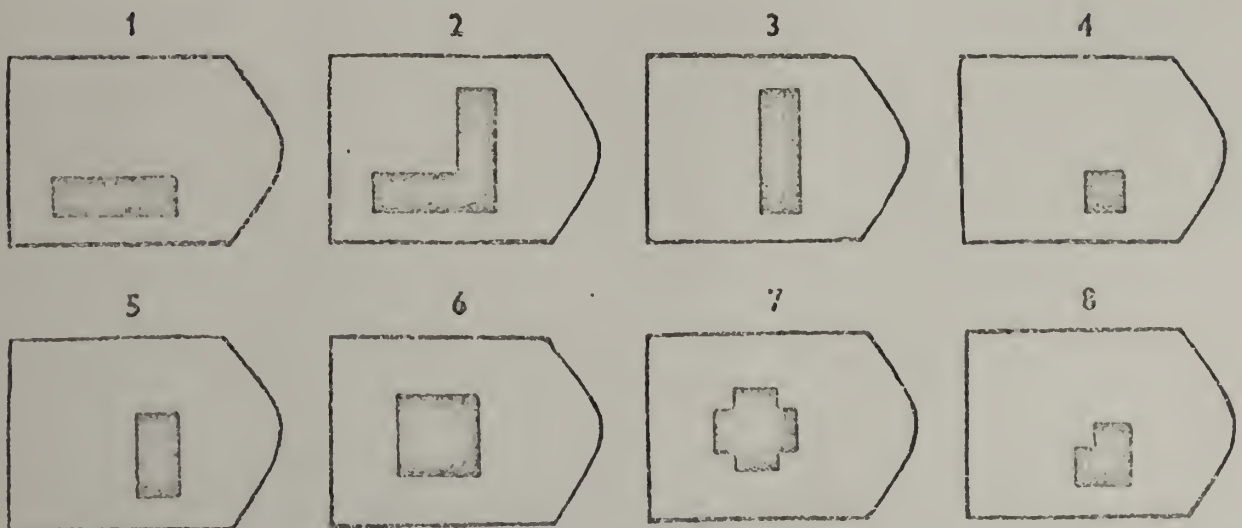
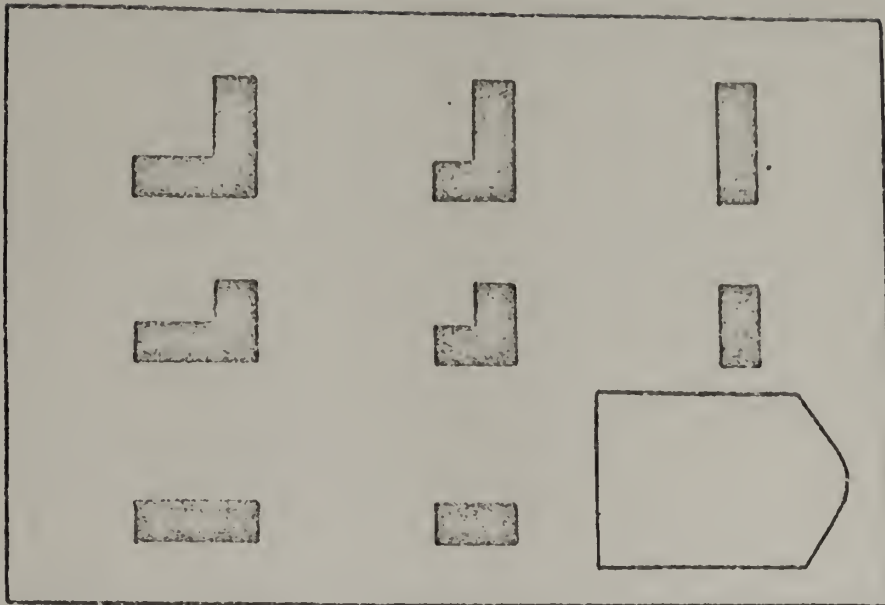
7



8

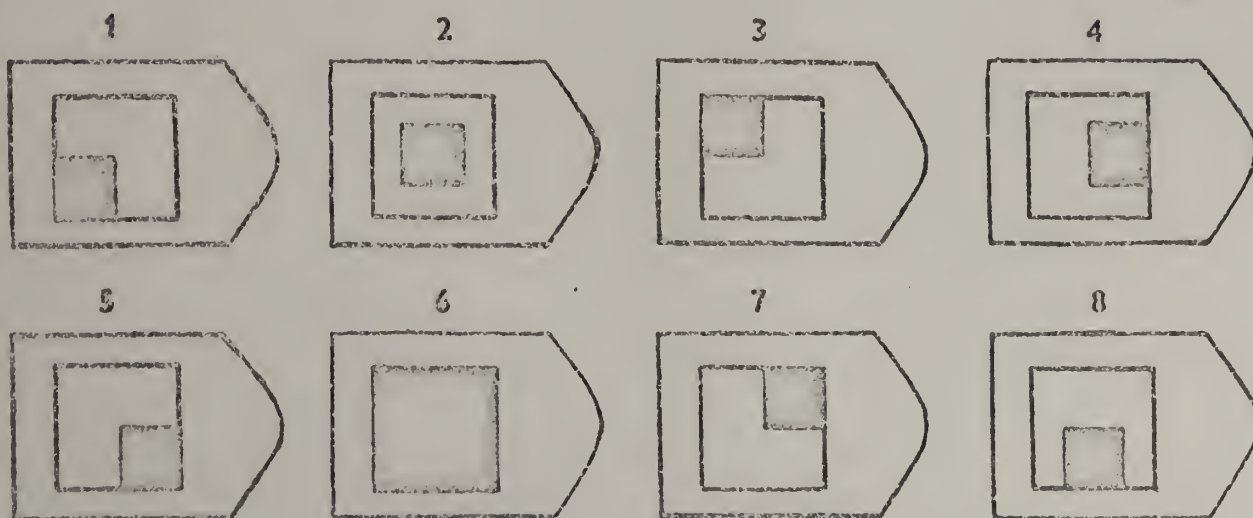
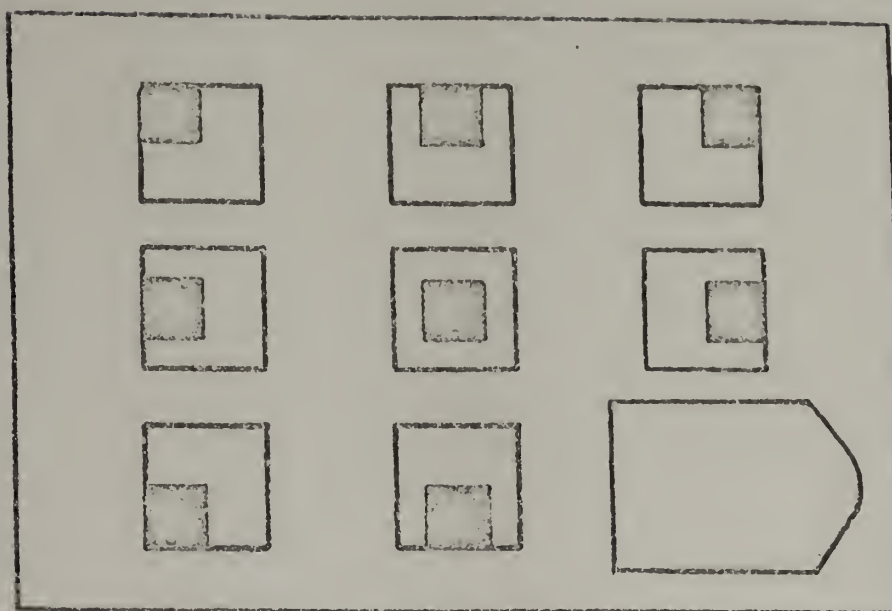


C6

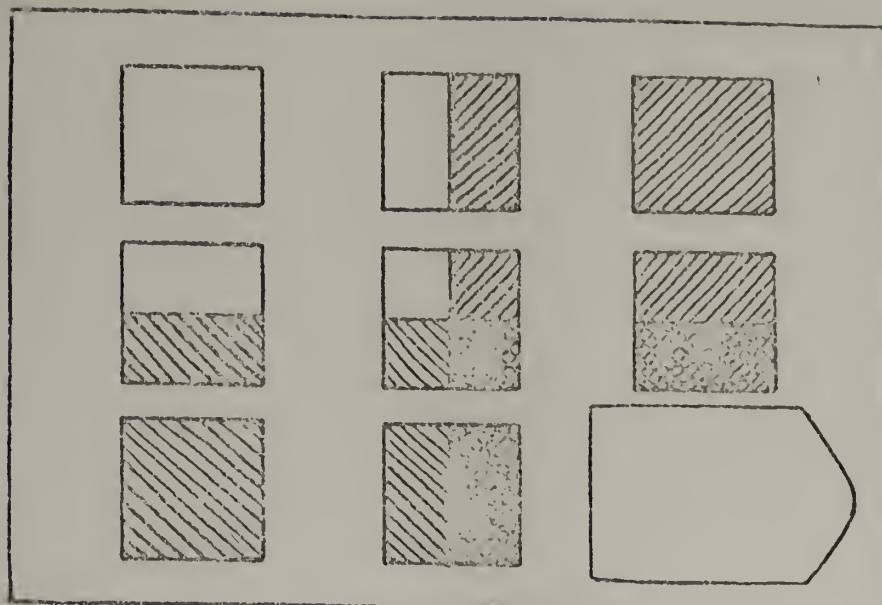




C7



C8



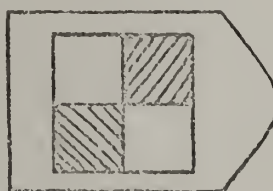
1



2



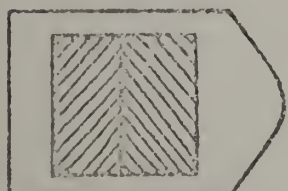
3



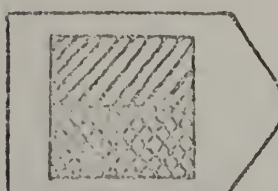
4



5



6



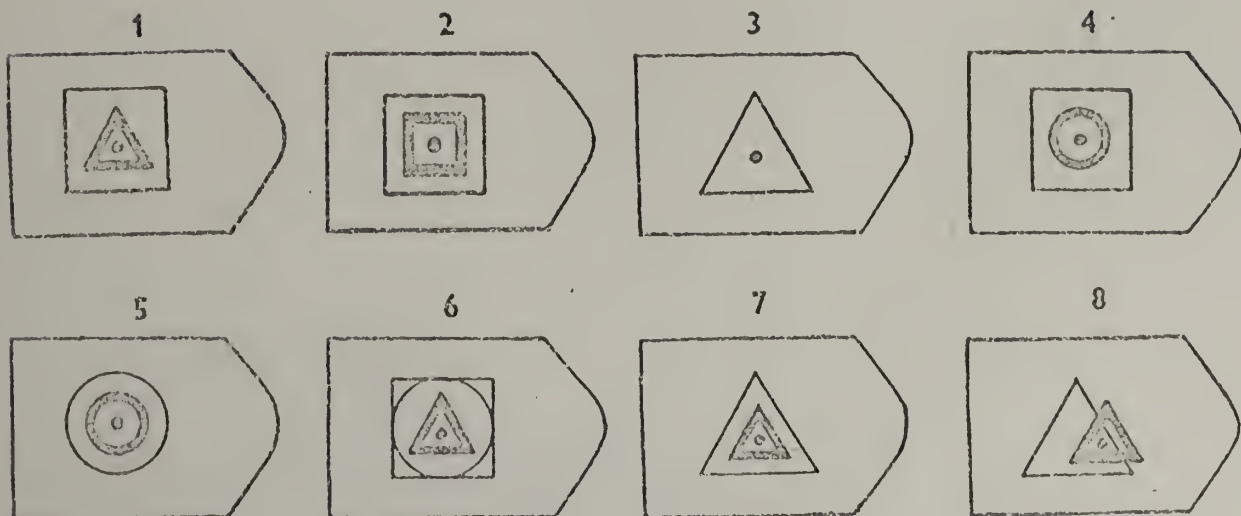
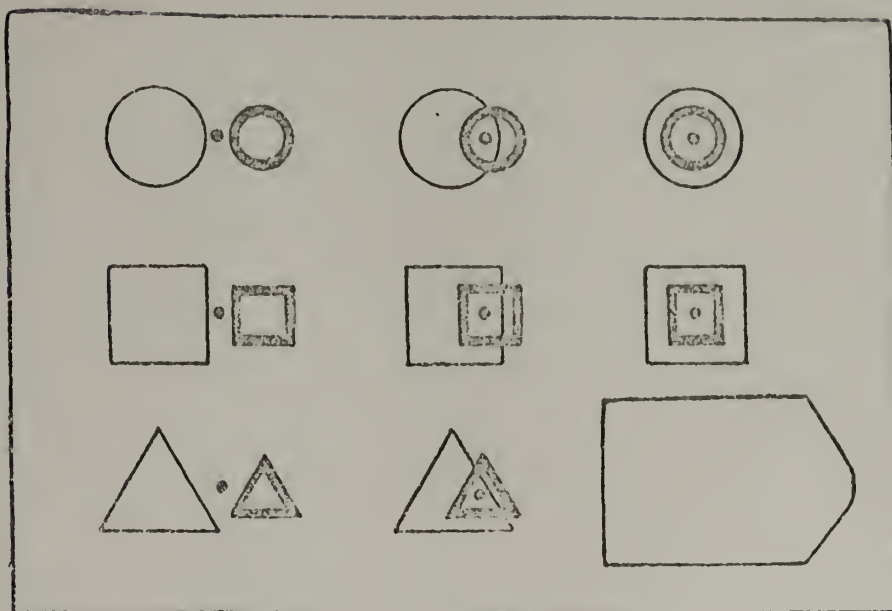
7

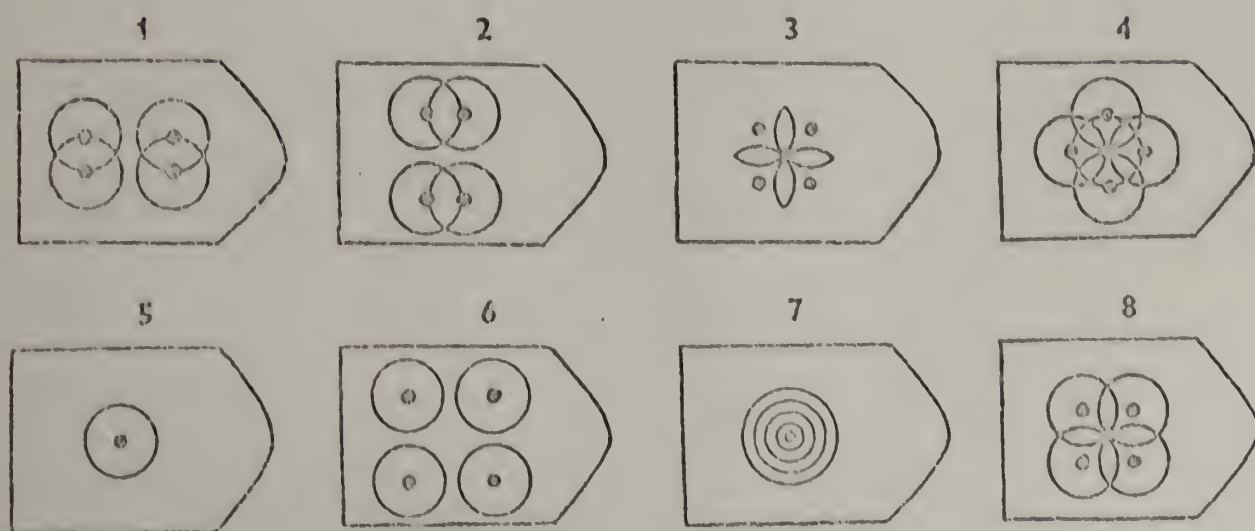
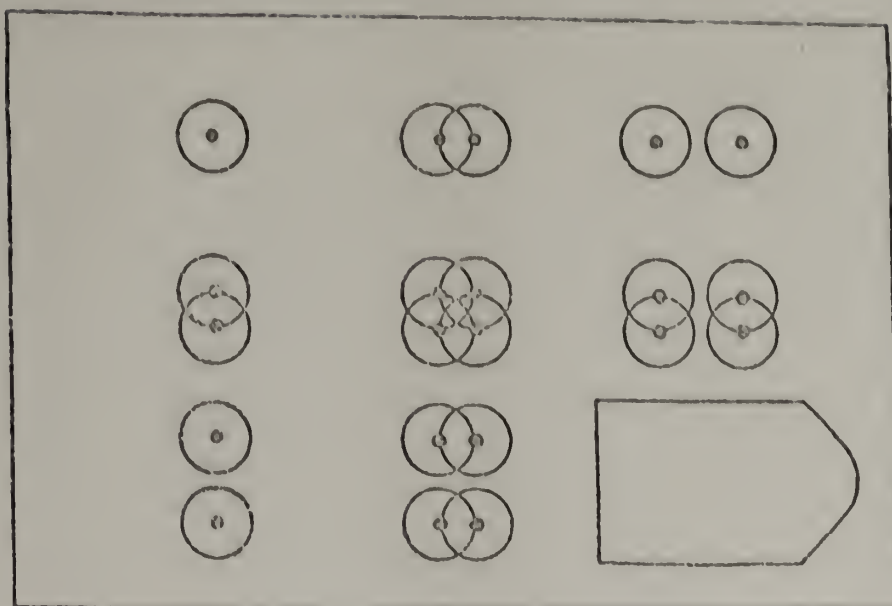


8

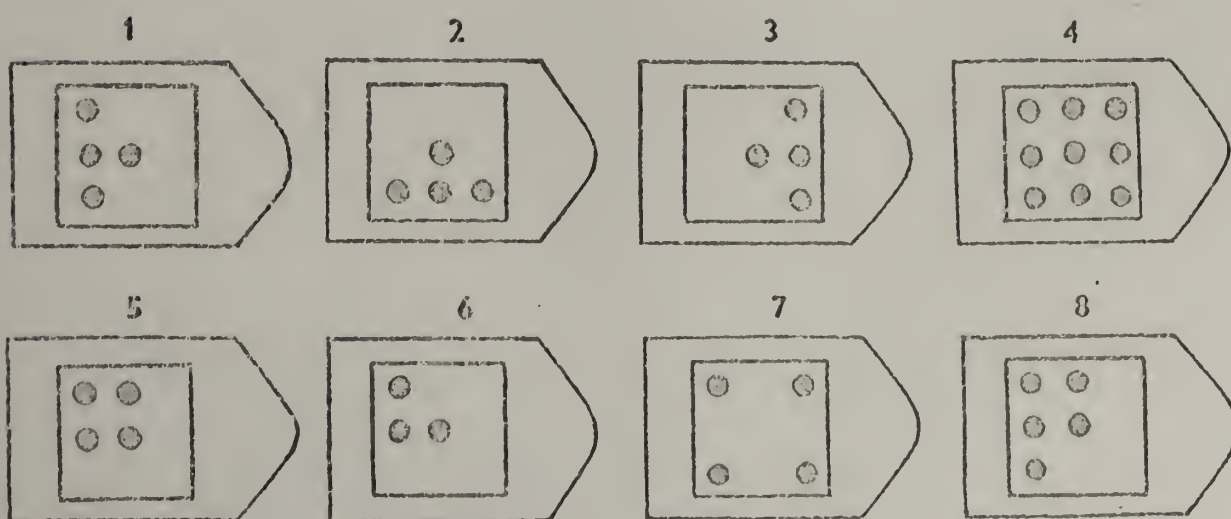
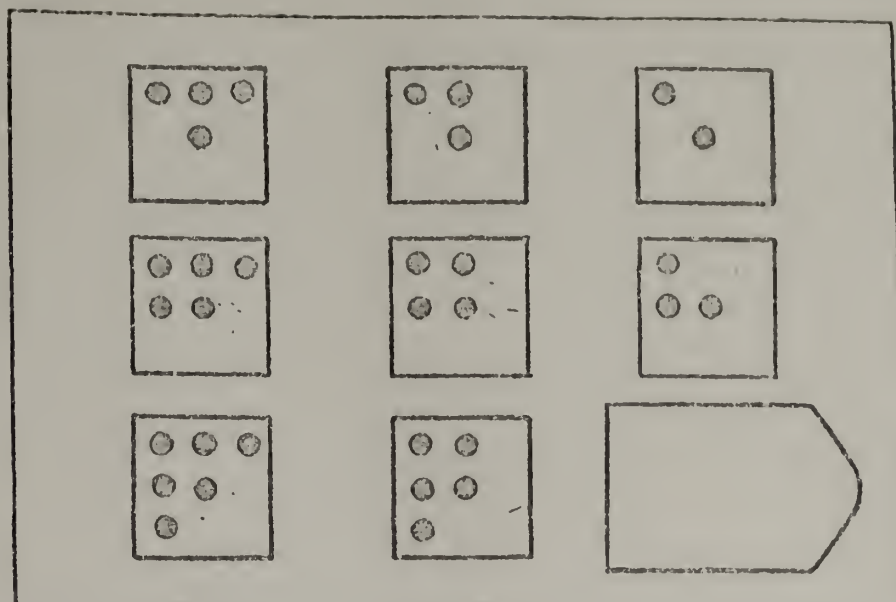


C9

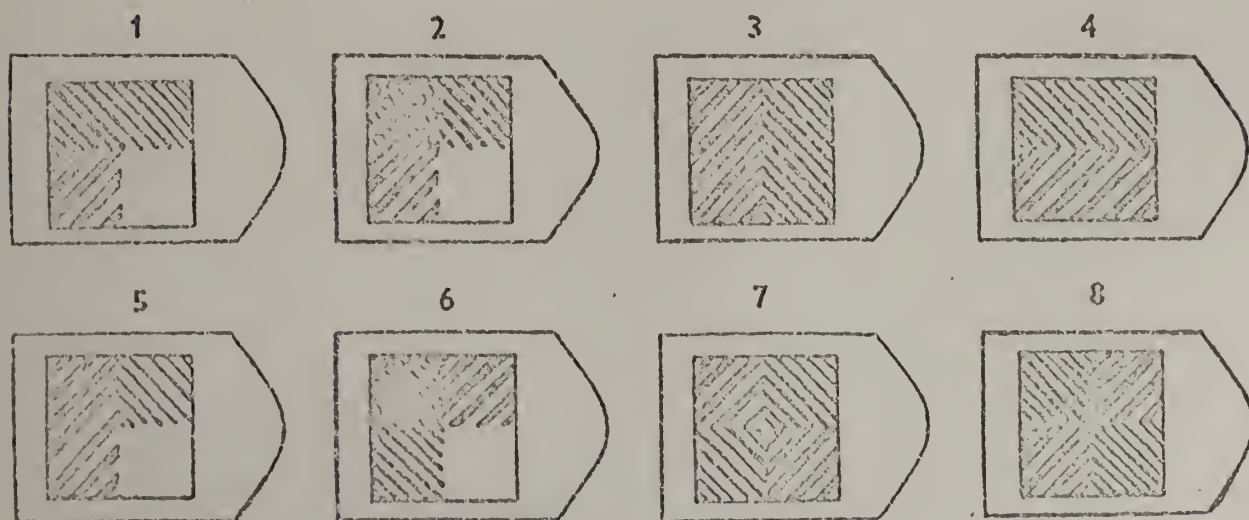
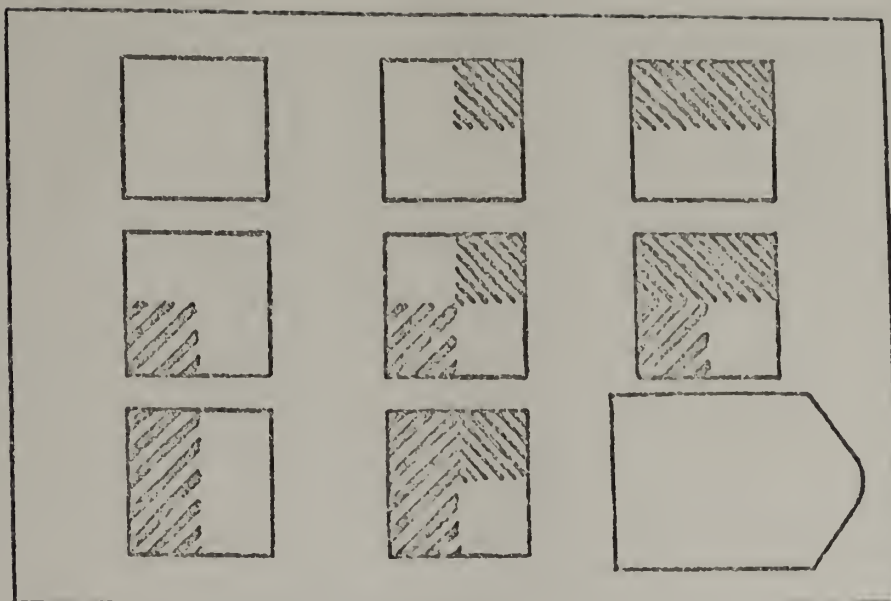


C<sub>10</sub>

## CII



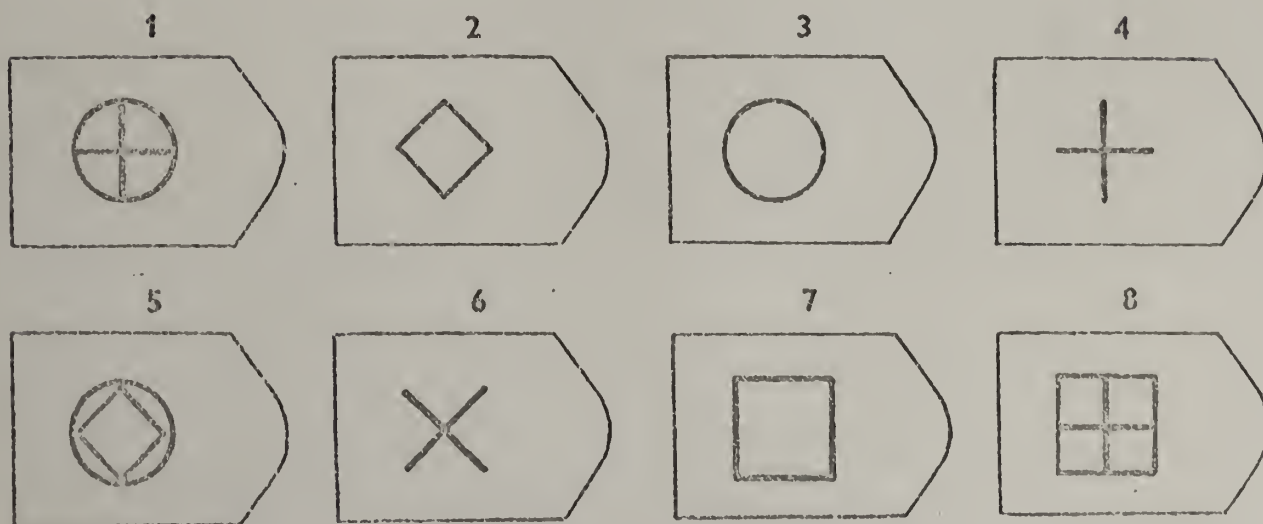
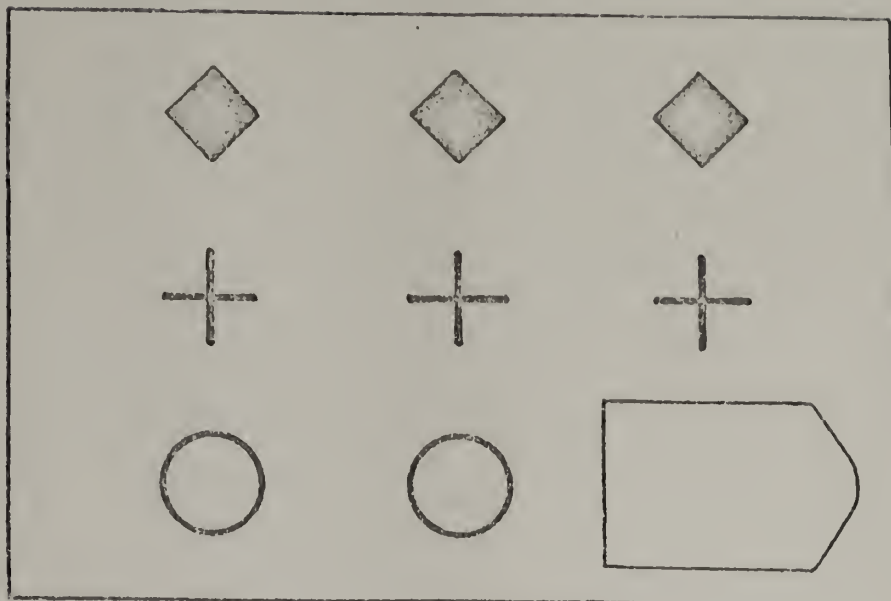


C<sub>12</sub>

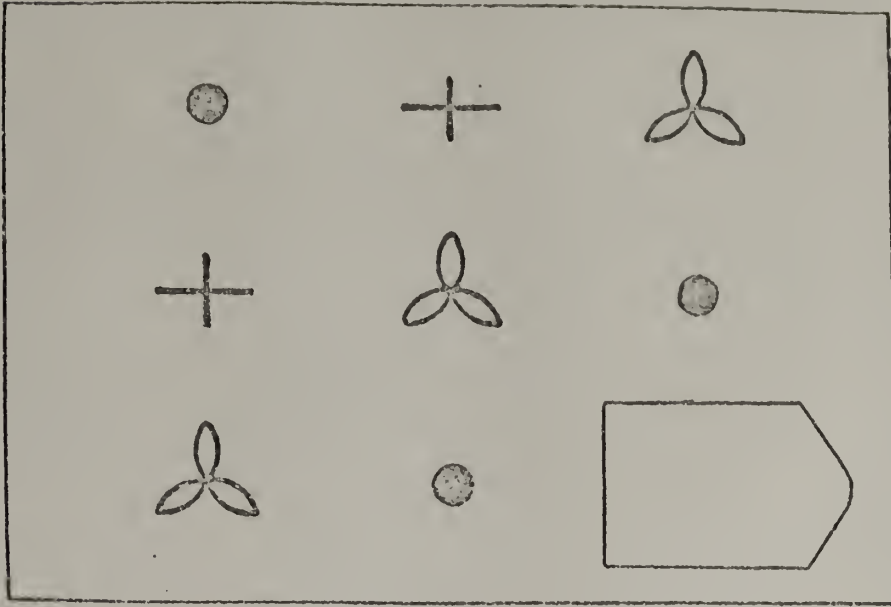


## SET D

D 1



D 2



1



2



3



4



5



6

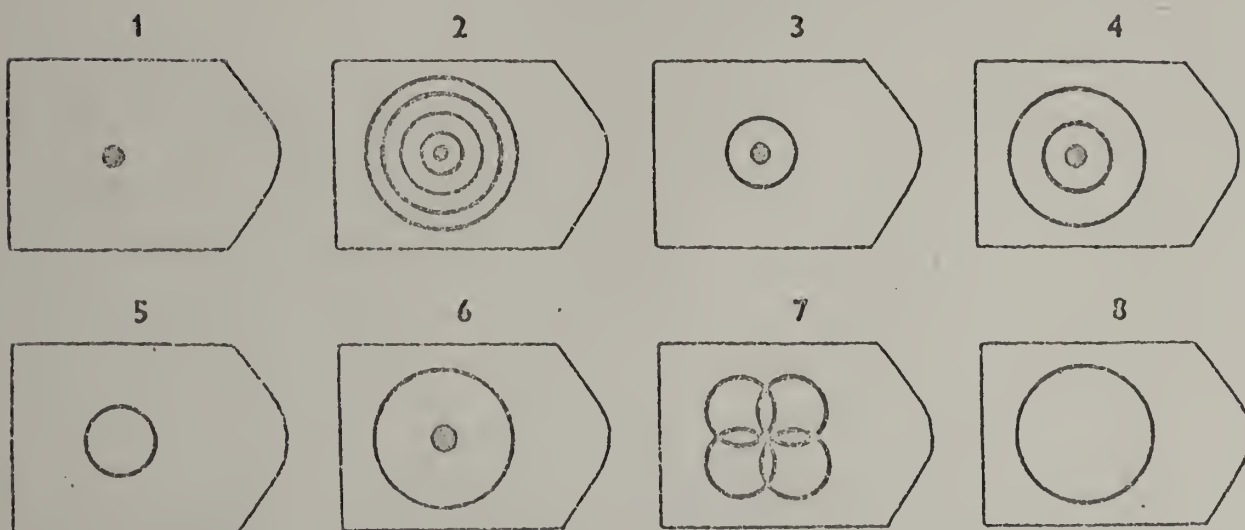
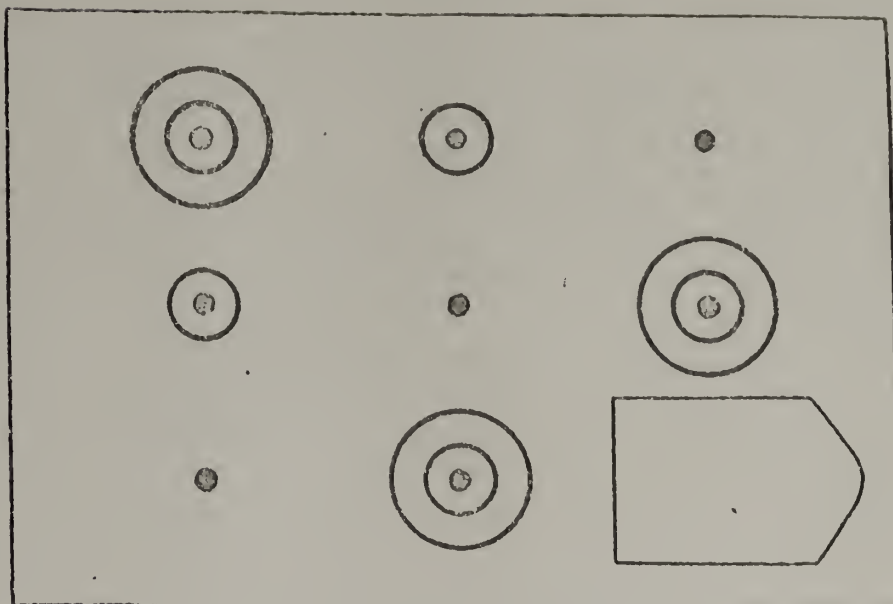


7

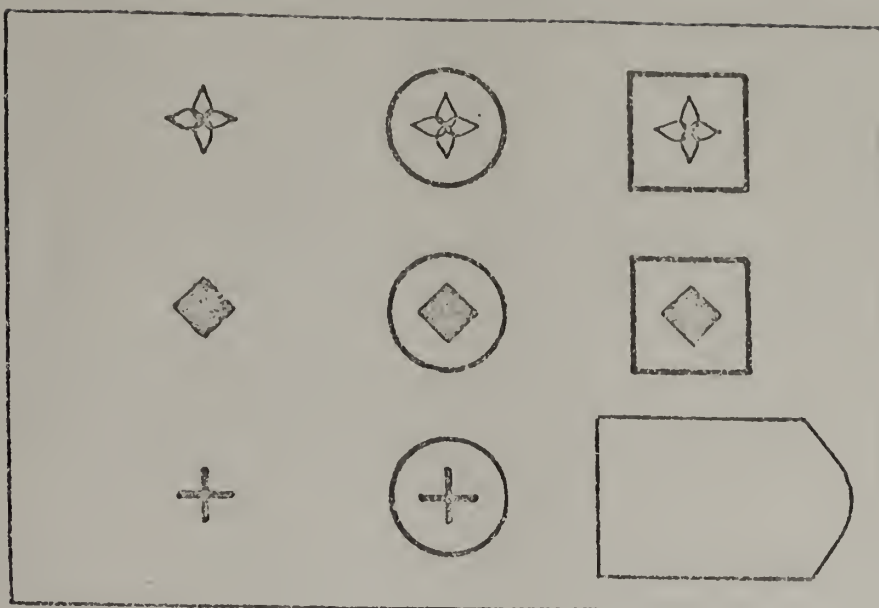


8

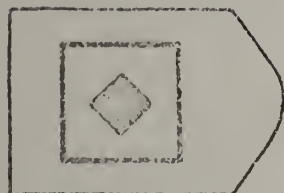


D<sub>3</sub>

D 4



1



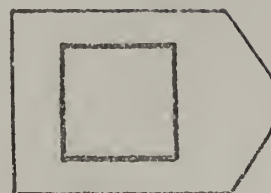
2



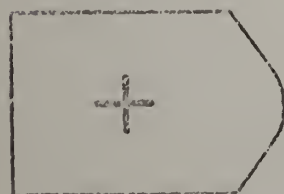
3



4



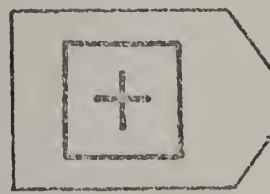
5



6



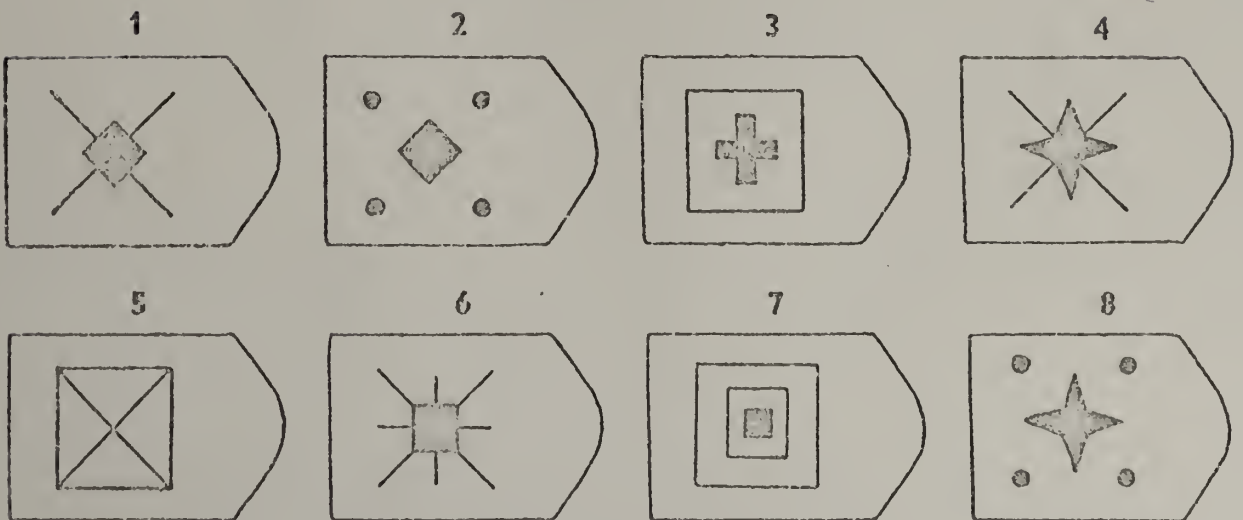
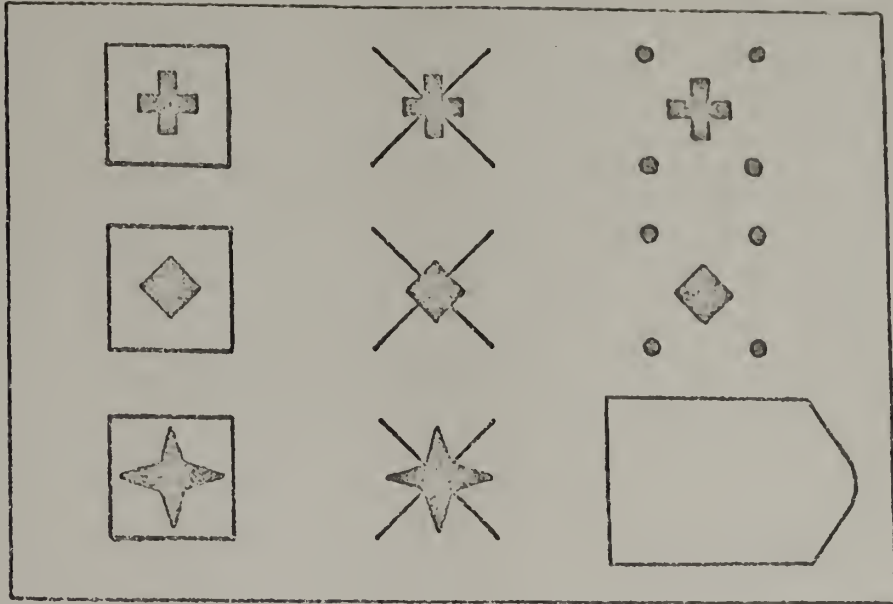
7



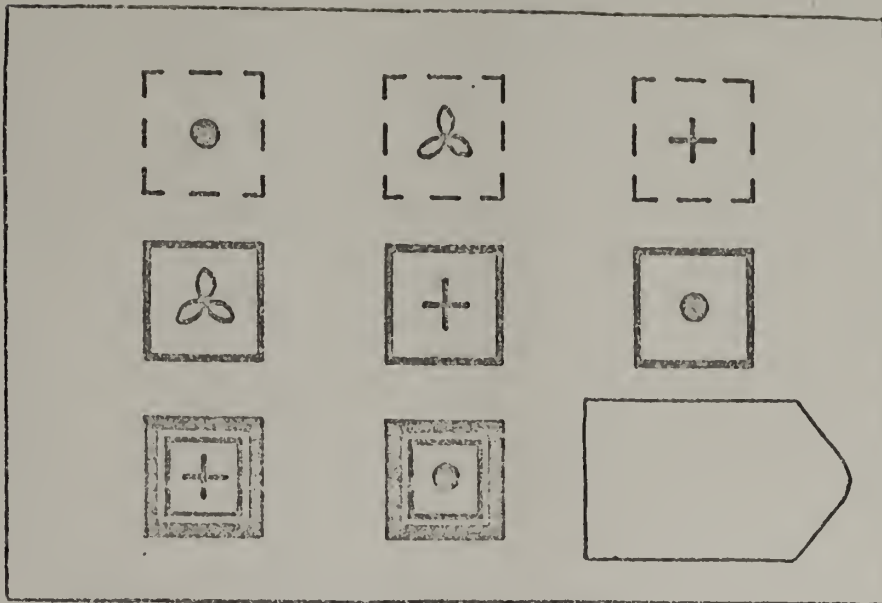
8



D<sub>5</sub>



D6



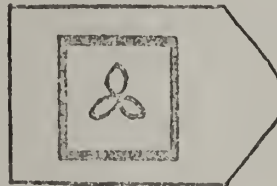
1



2



3



4



5



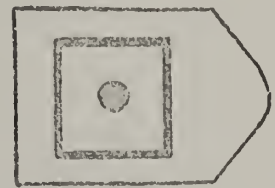
6



7

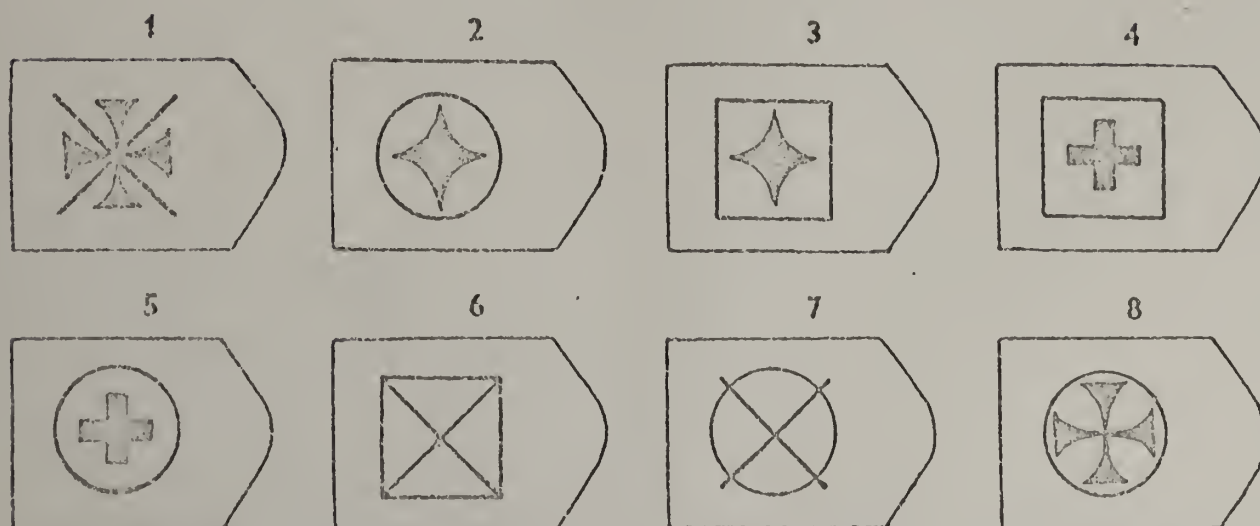
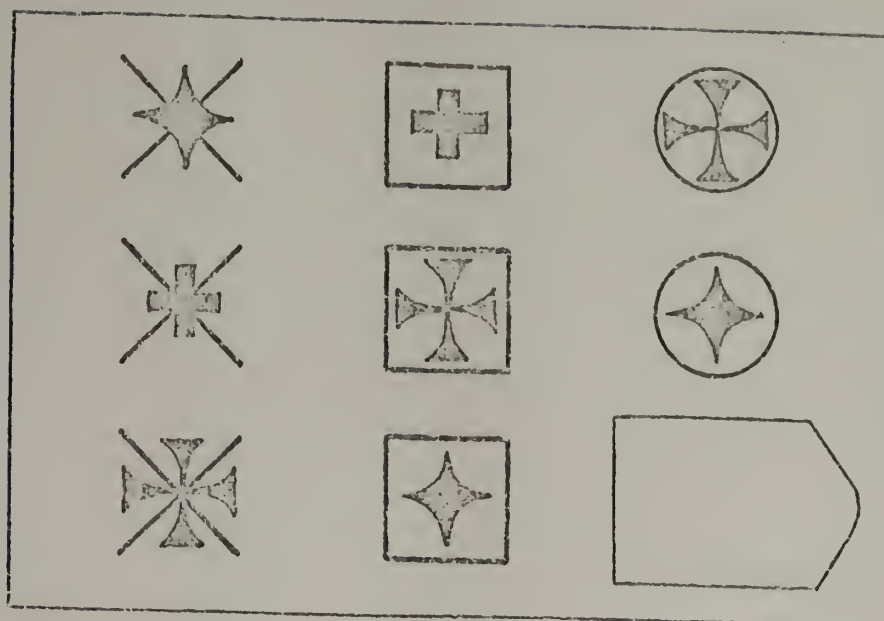


8

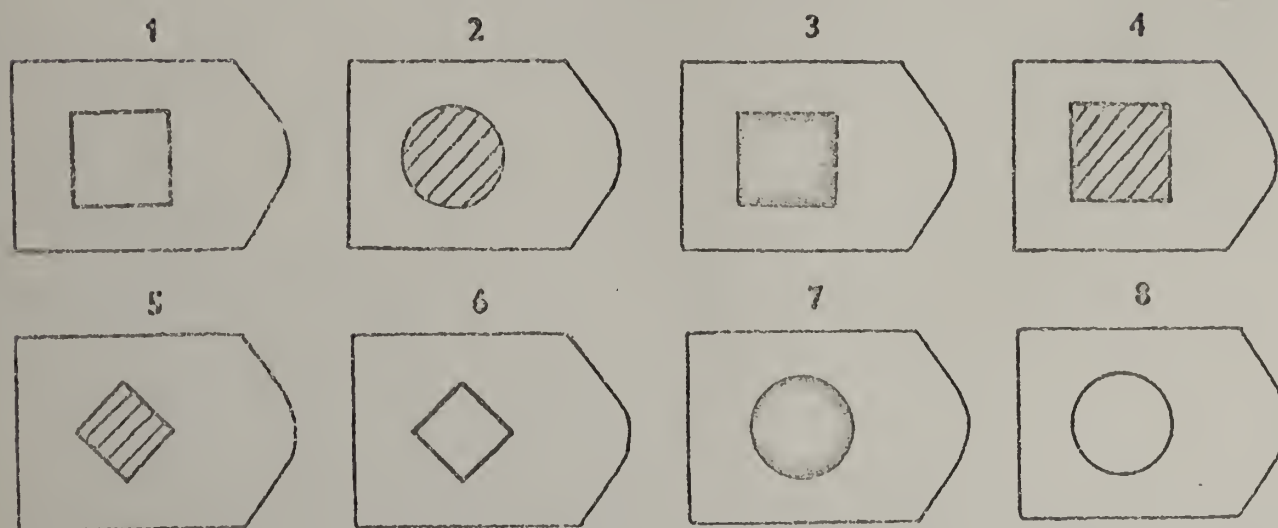
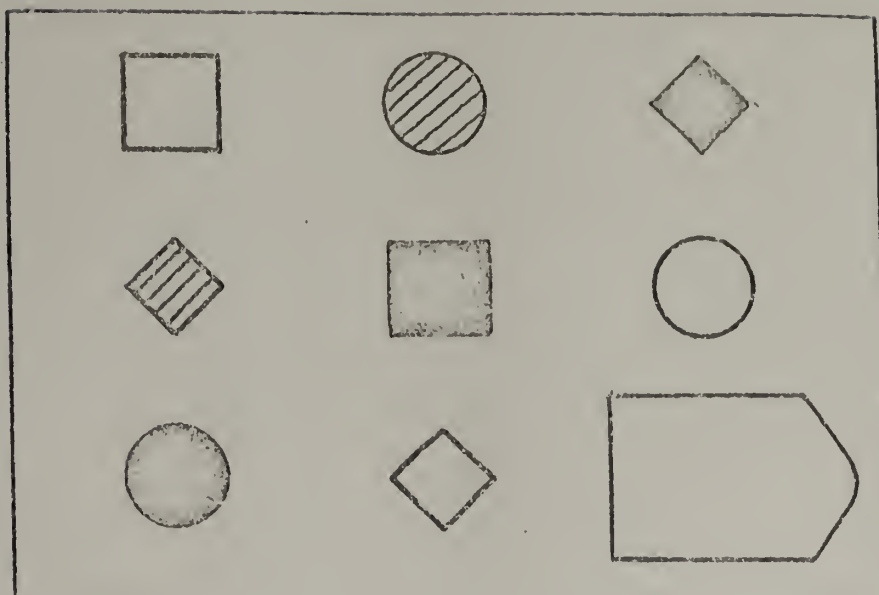




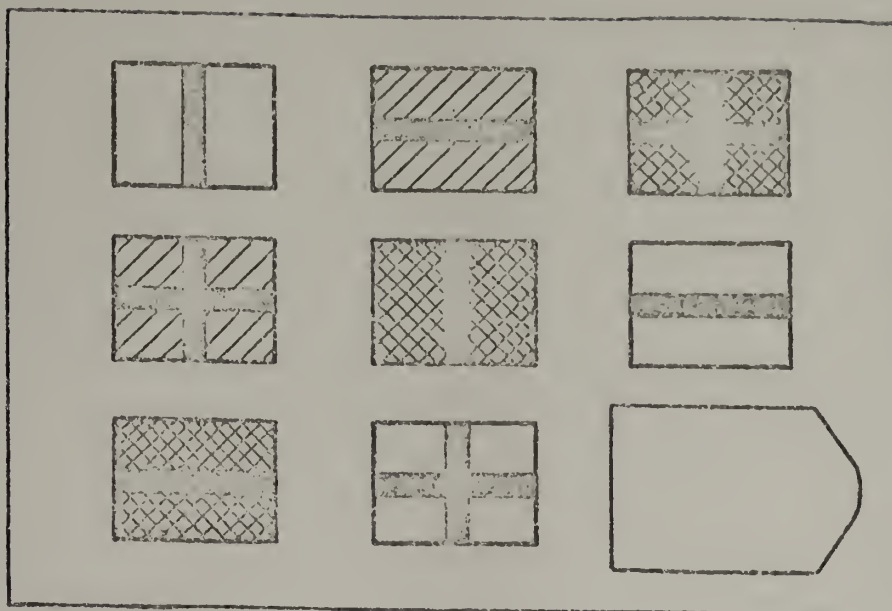
D7



D 8



D9

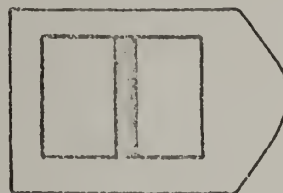
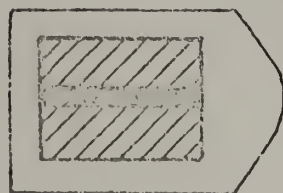


1

2

3

4

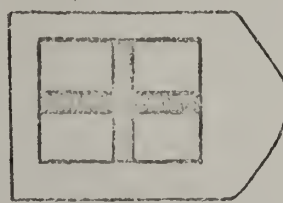
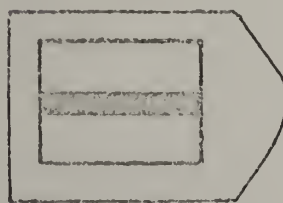
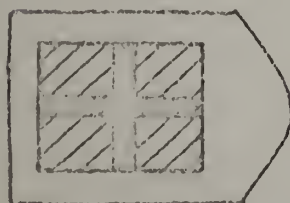


5

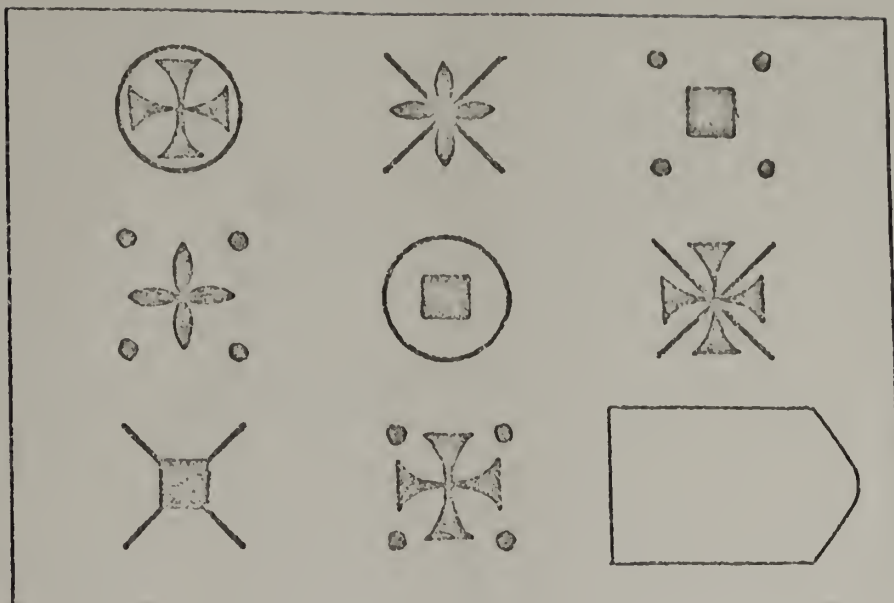
6

7

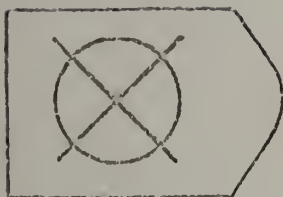
8



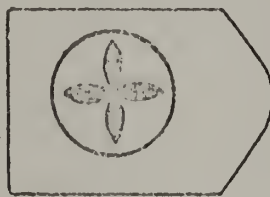
D 10



1



2



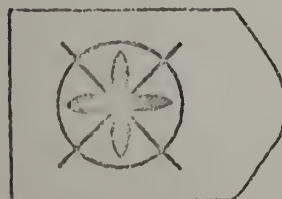
3



4



5



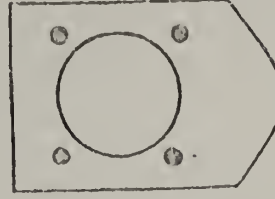
6



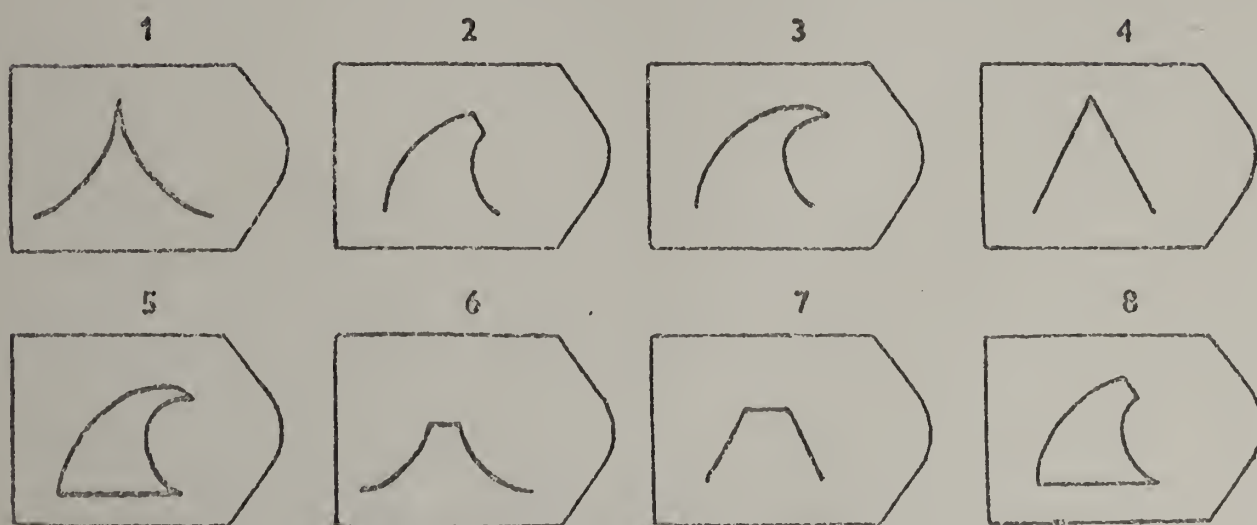
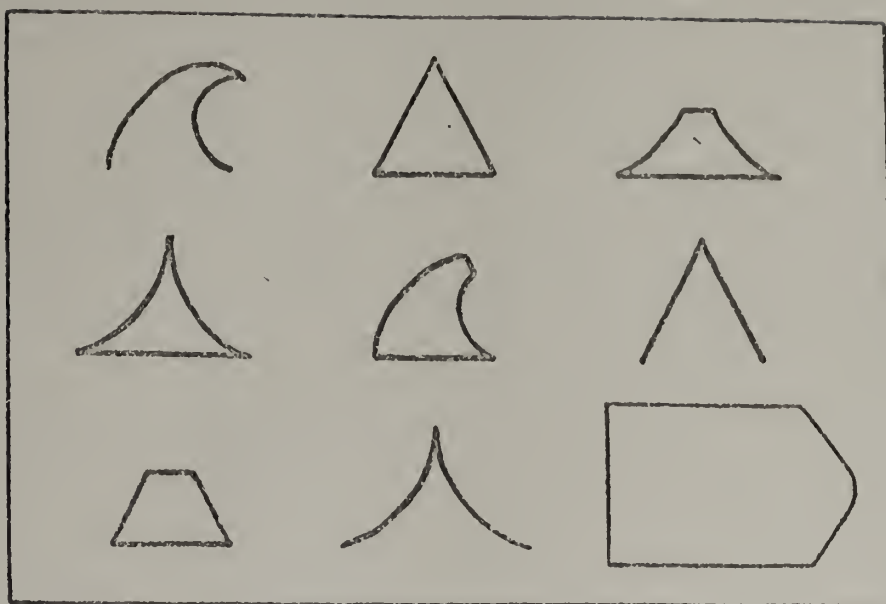
7



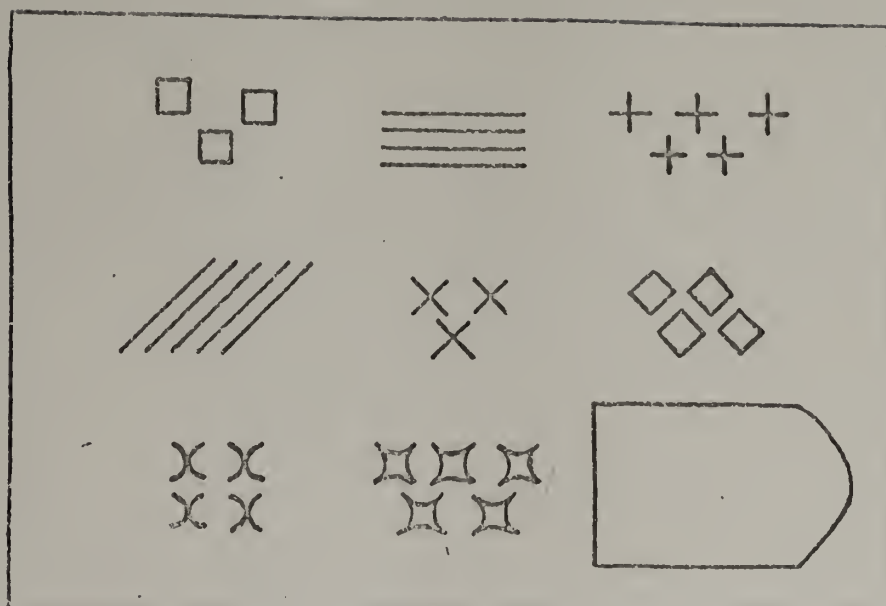
8



DIR



D 12



1



2



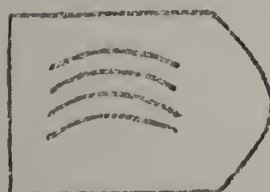
3



4



5



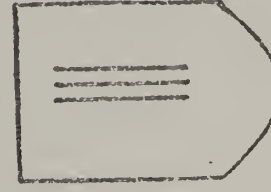
6



7



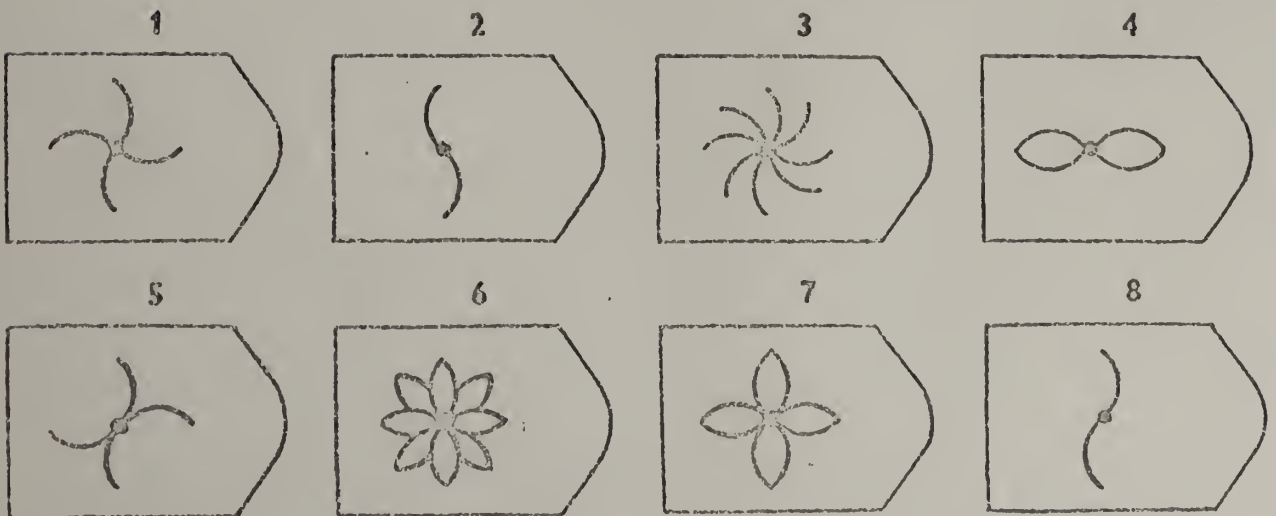
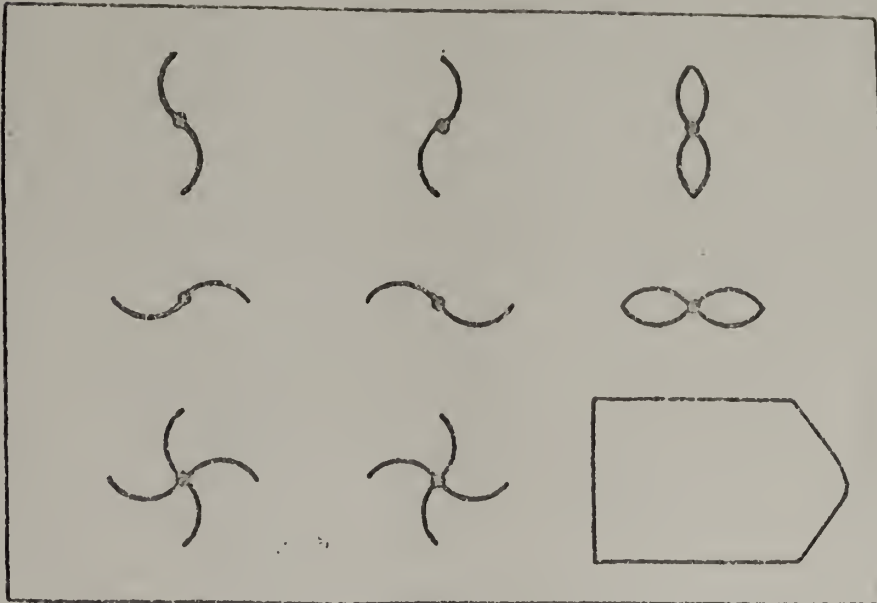
8



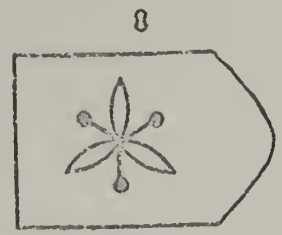
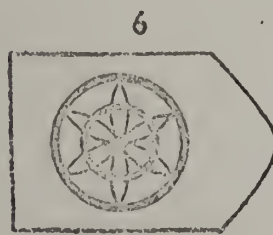
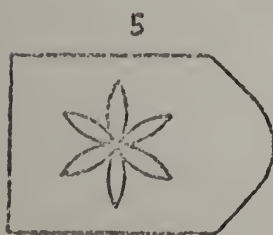
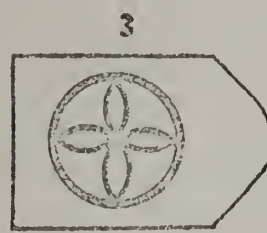
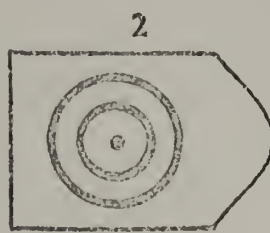
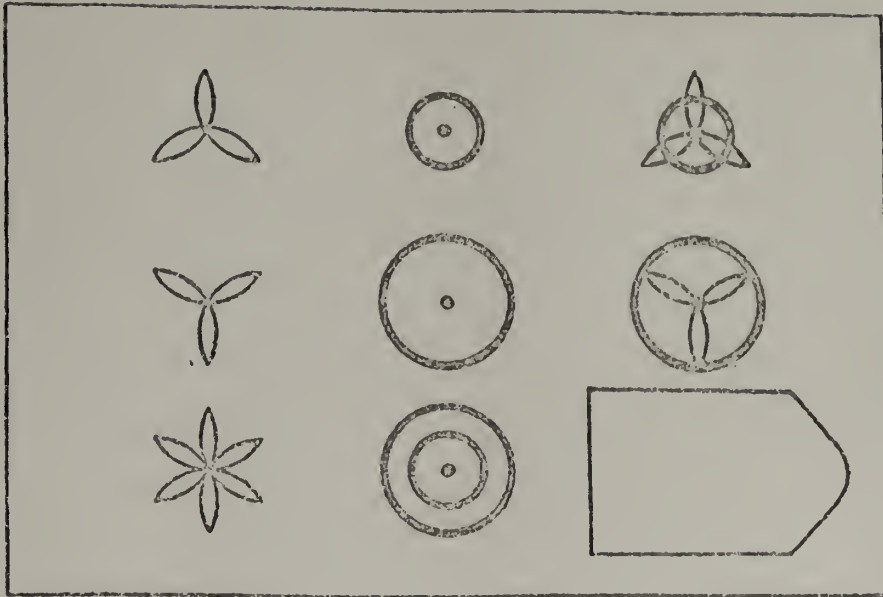


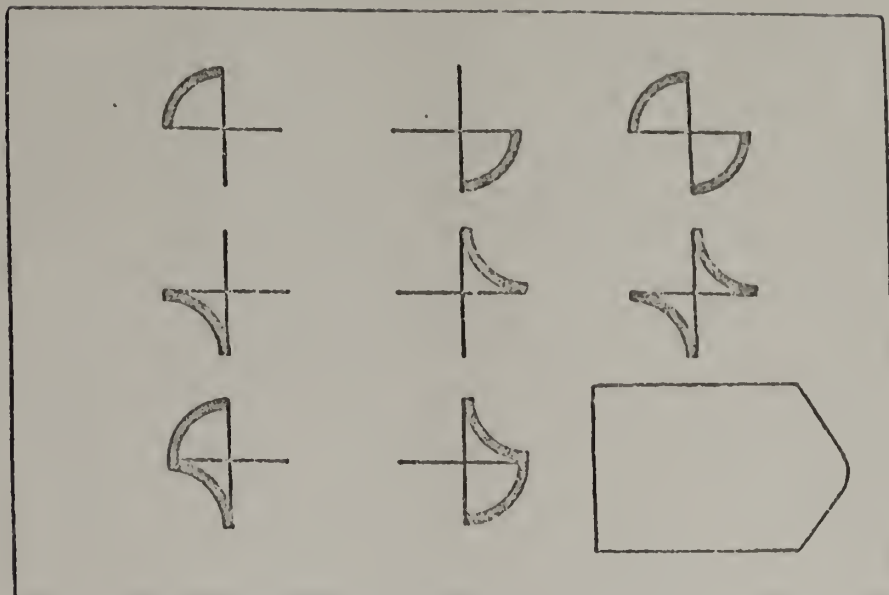
## SET E

E I

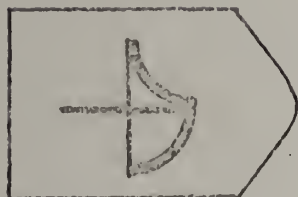


E 2

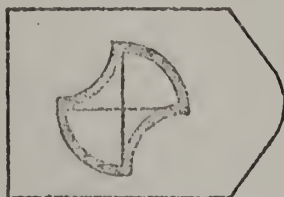


E<sub>3</sub>

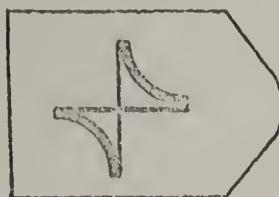
1



2



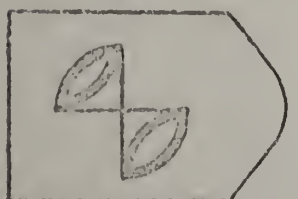
3



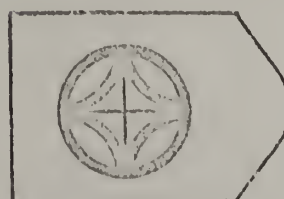
4



5



6

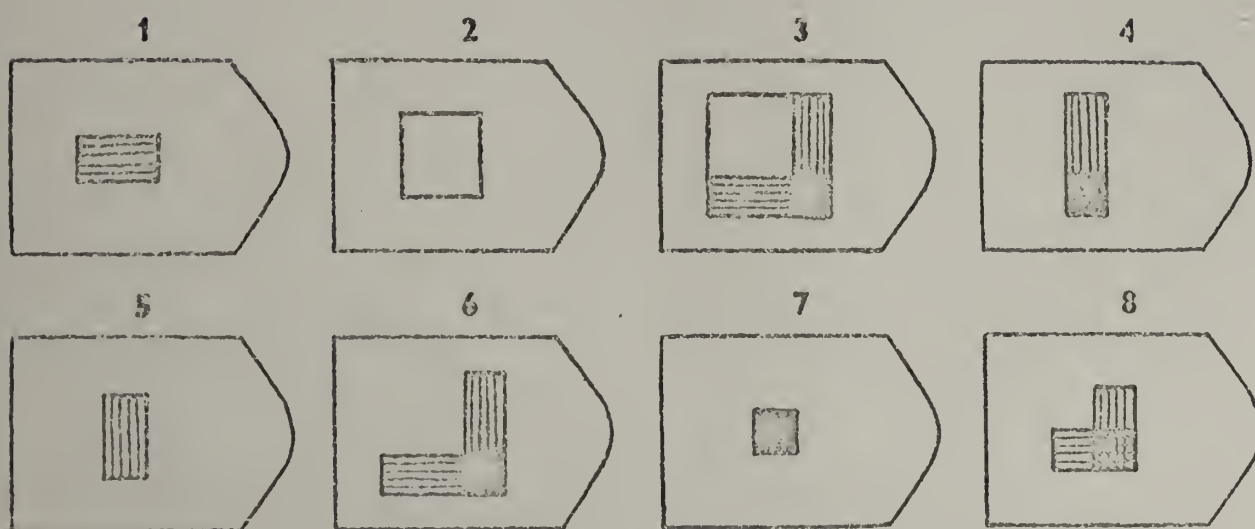
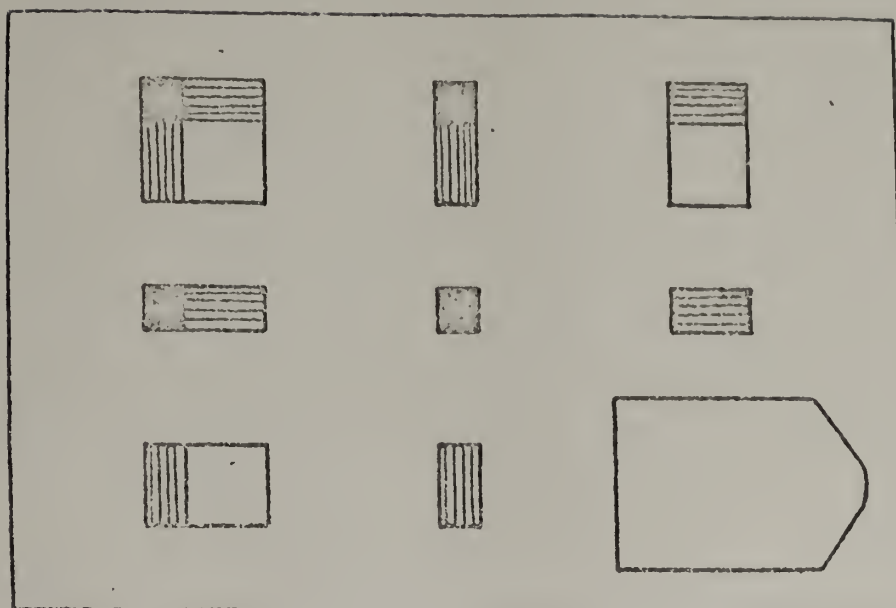


7



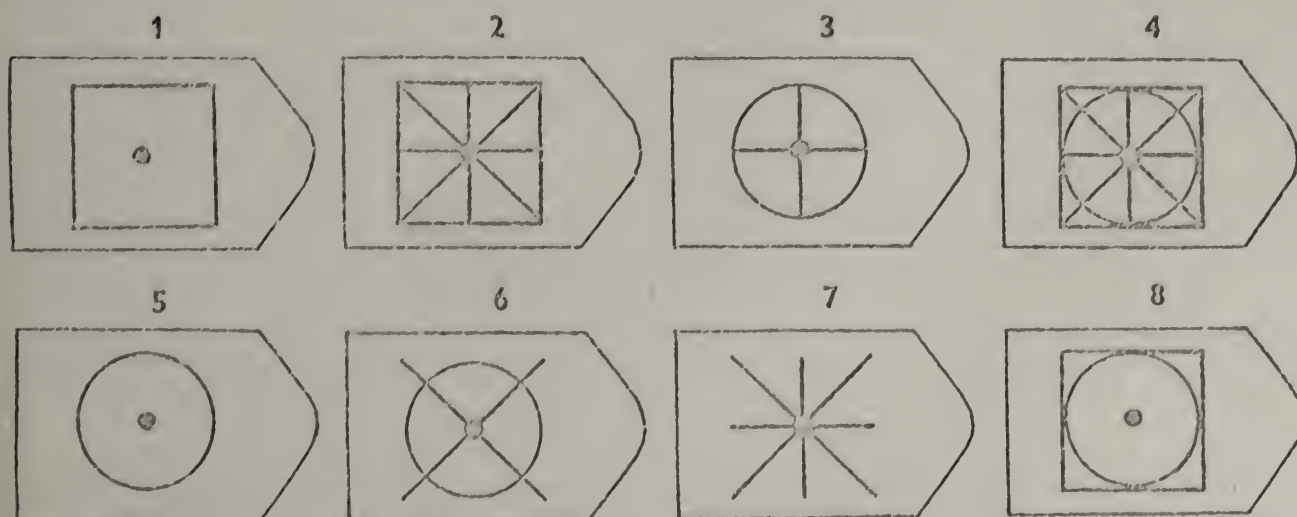
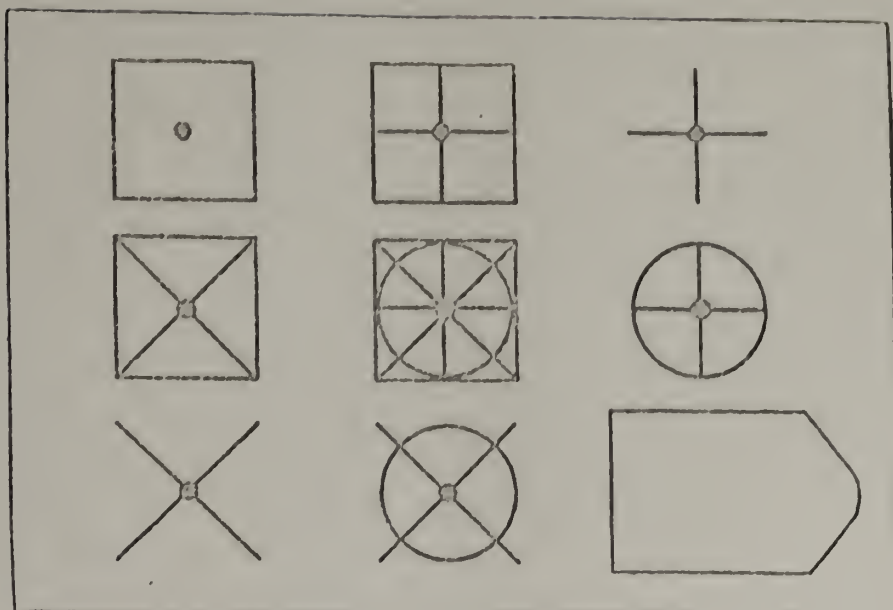
8



E<sub>4</sub>

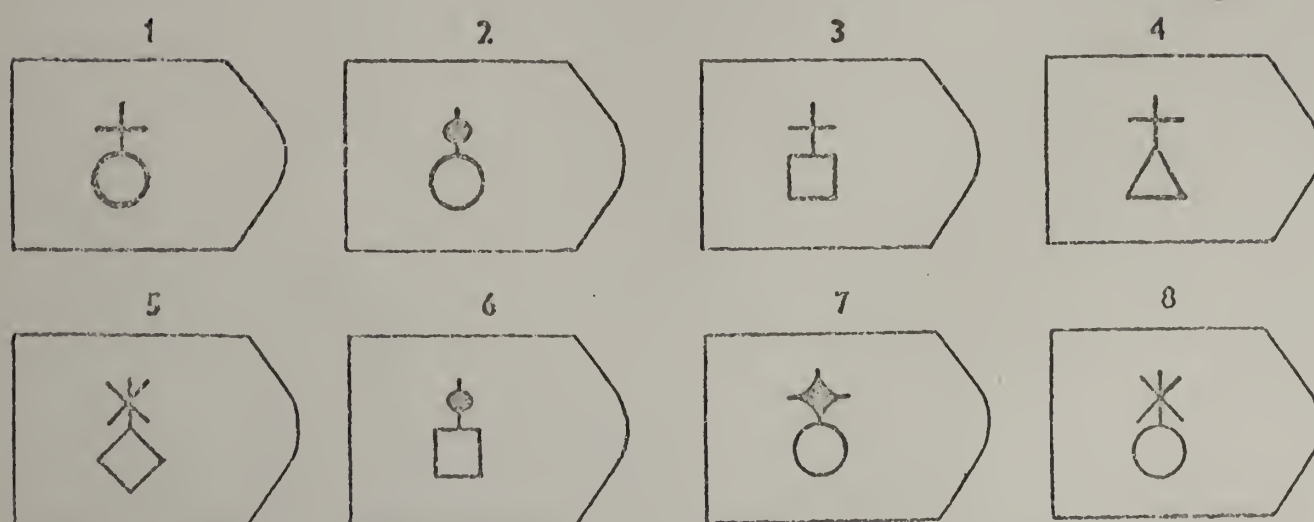
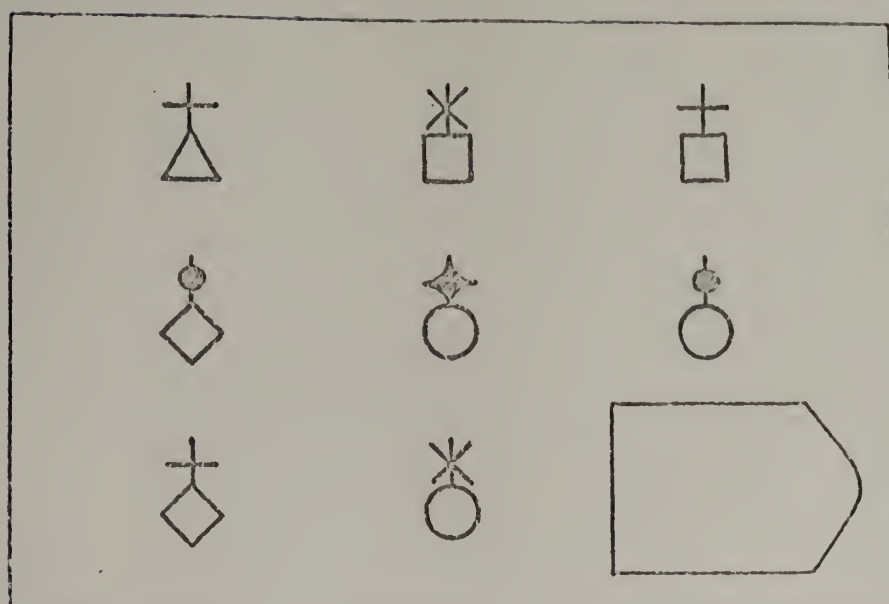


E6

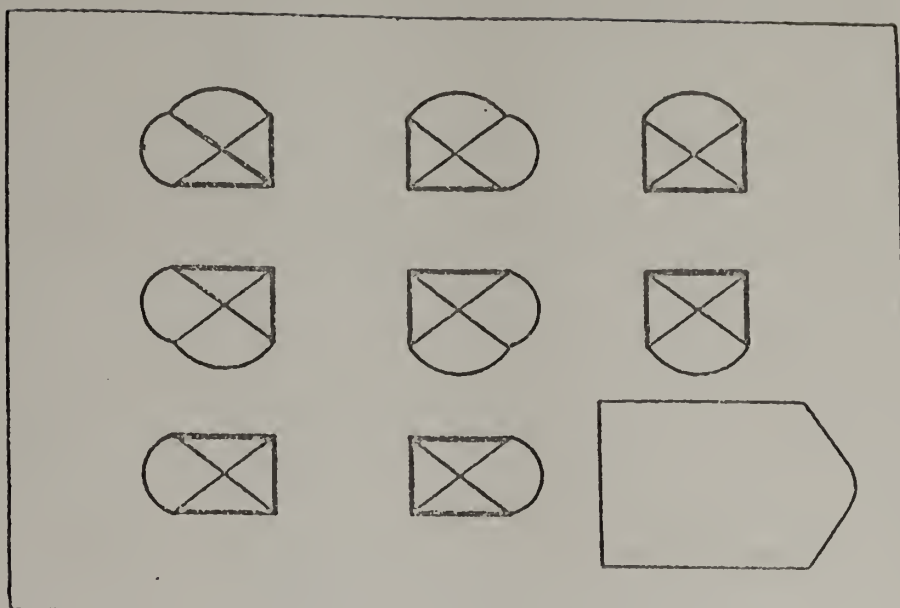




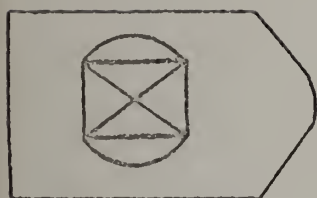
E7



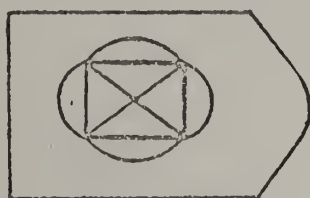
E 8



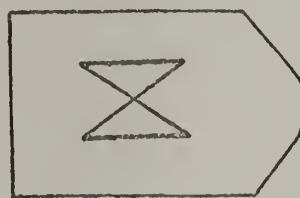
1



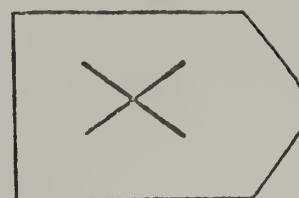
2



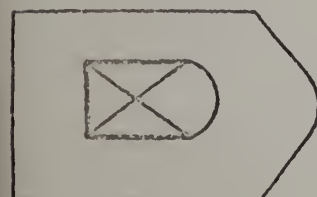
3



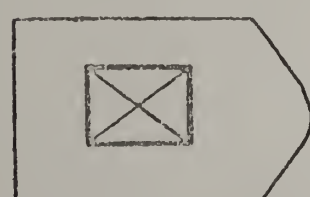
4



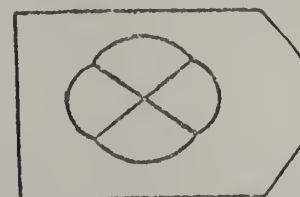
5



6



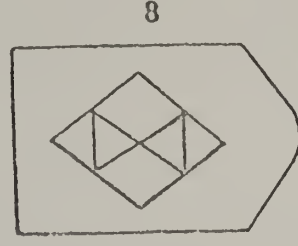
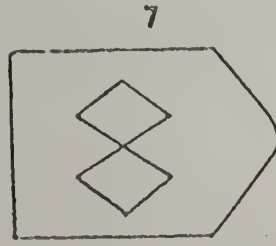
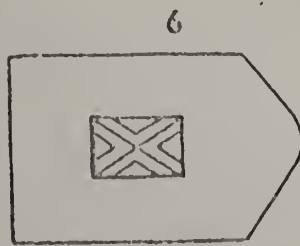
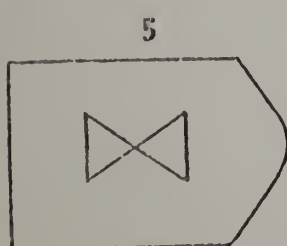
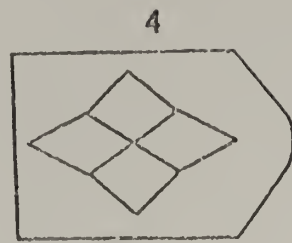
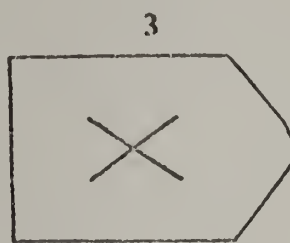
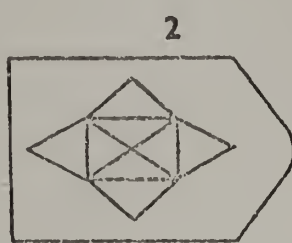
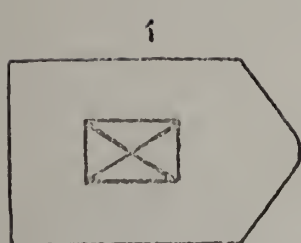
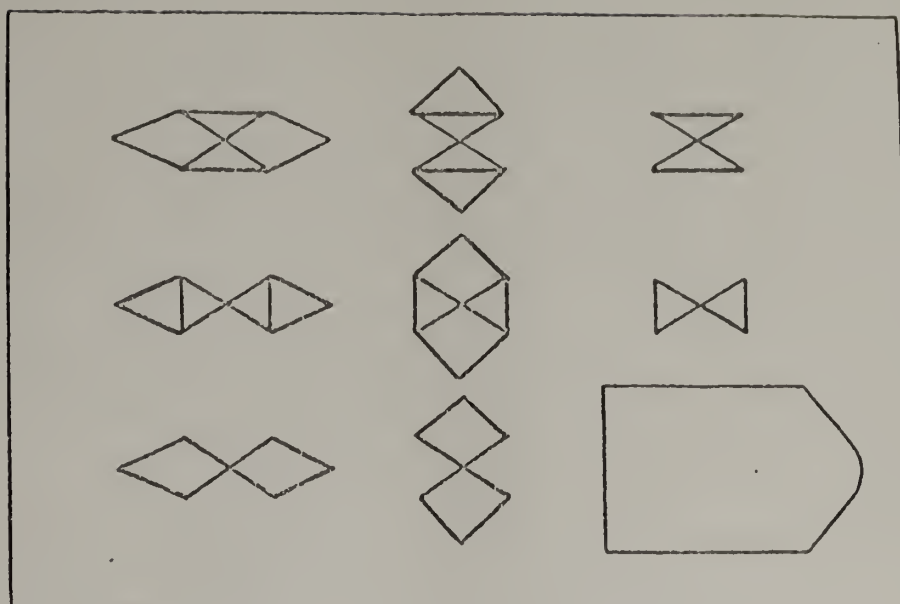
7



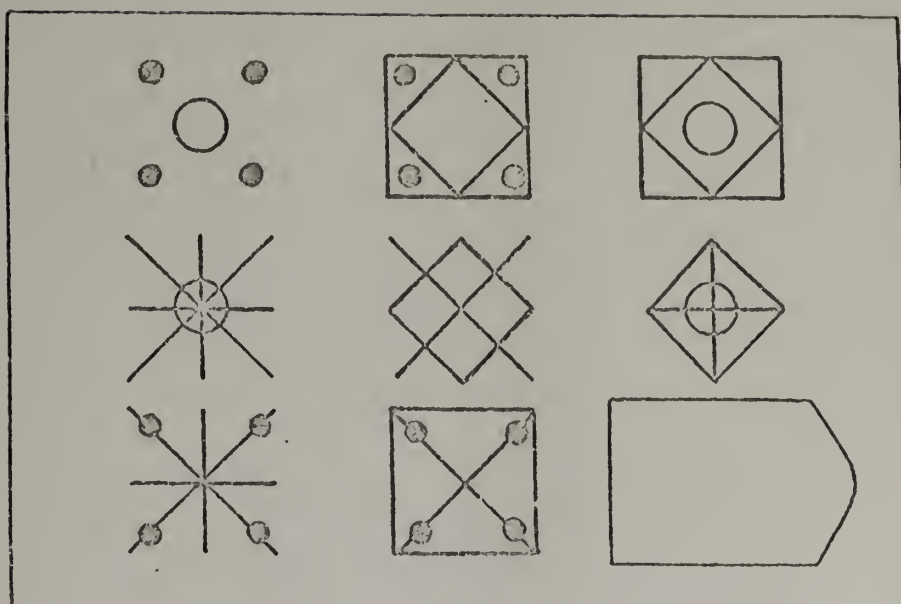
8



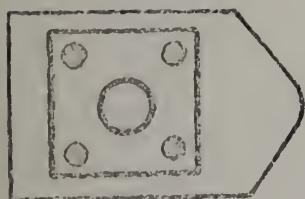
E9



Его



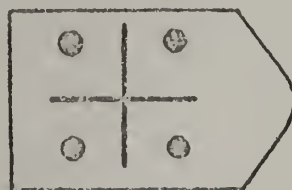
1



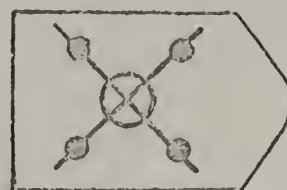
2



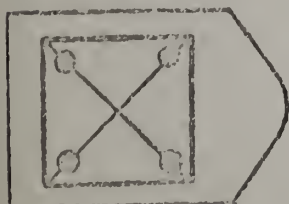
3



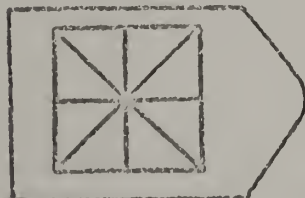
4



5



6



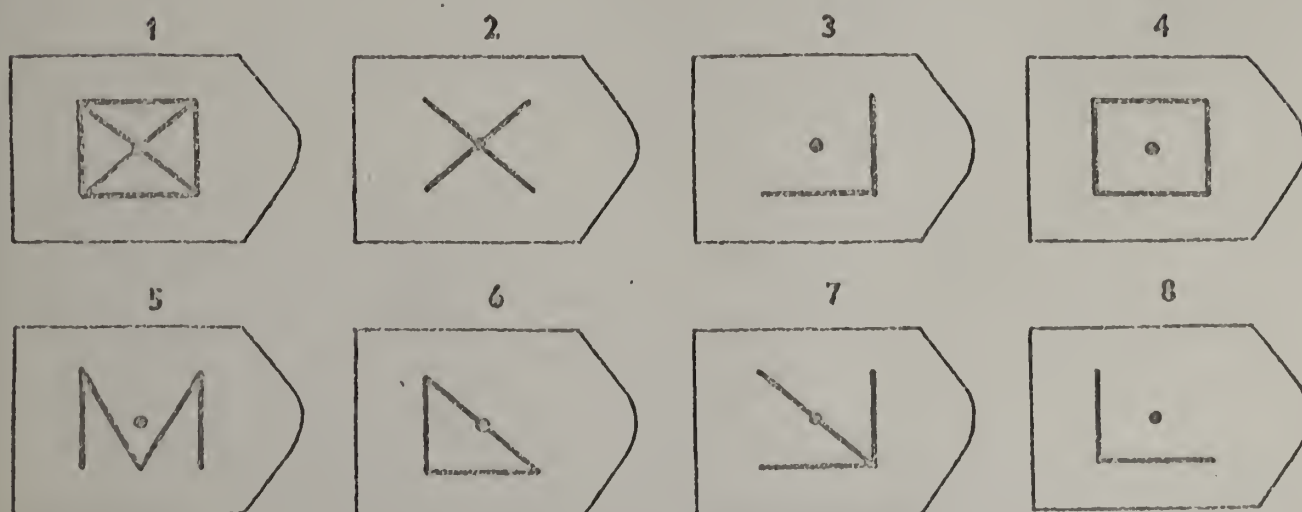
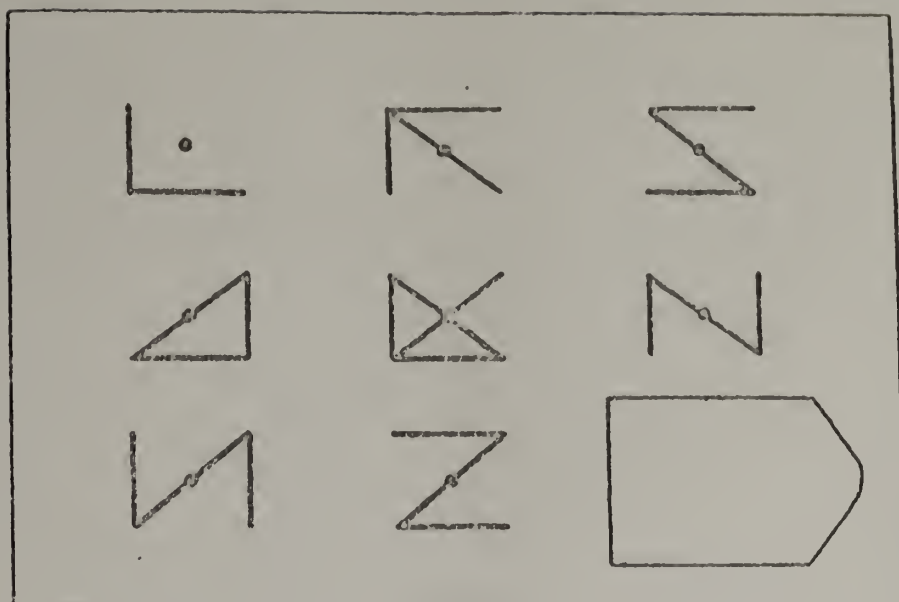
7



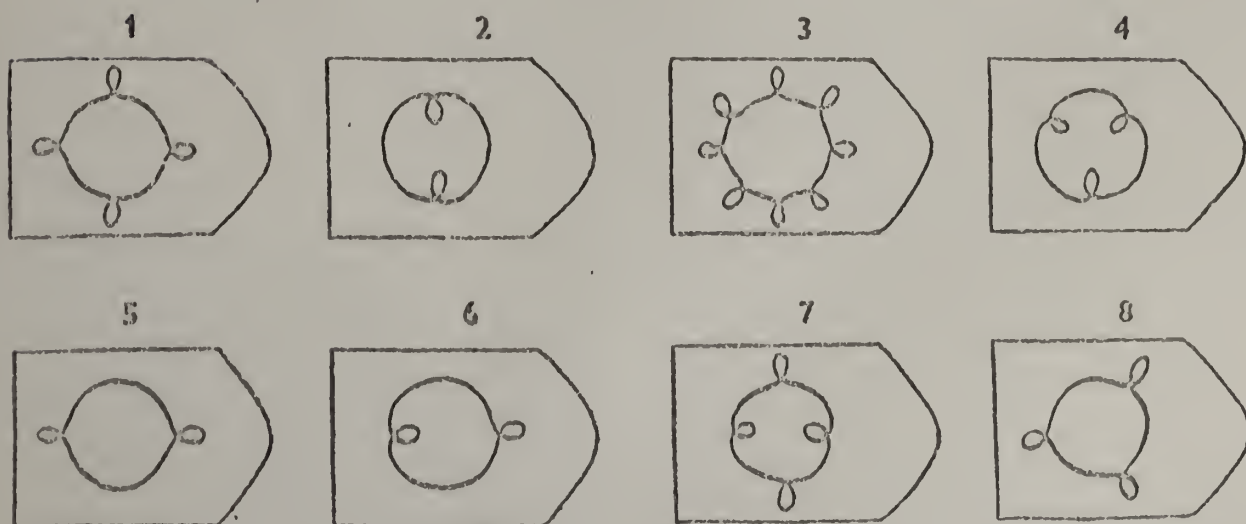
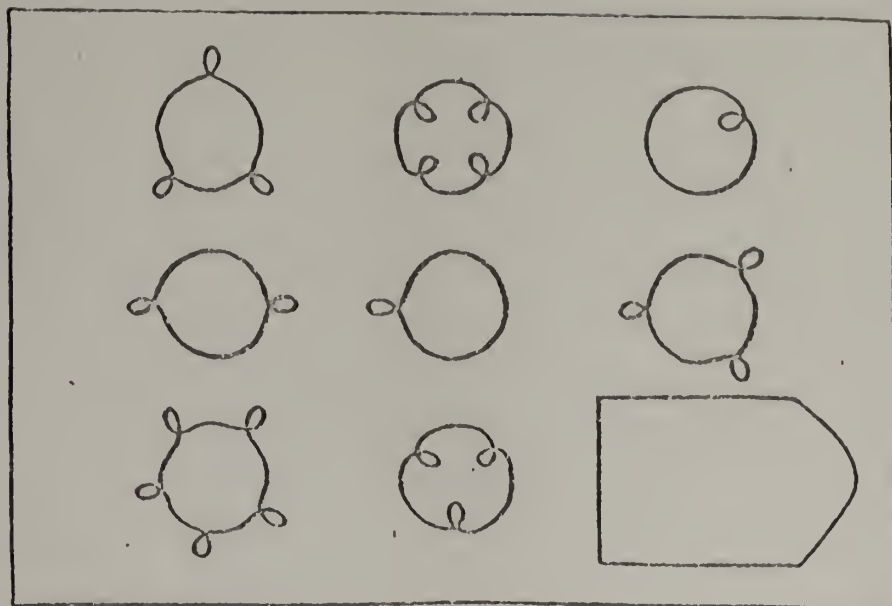
8



## E II



E 12





## APPENDIX C

## MODELING ACHIEVEMENT TEST

NAME: \_\_\_\_\_

Follow directions carefully. In the multiple choice questions, pick the one best answer.

1. Which of the following statements about models is not true?
  - A. Models can be revised or changed.
  - B. Models describe or represent something.
  - C. Models are complete.
  - D. Data is used to develop models.
  
2. A game warden in charge of managing a deer herd on a state reservation wishes to increase the size of his herd by bringing in deer from another reservation. Before doing so, he builds a population growth model to see what effect the additional deer will have on the size of the herd in years to come. What is the main advantage to building such a model?
  - A. The warden will not have to take a yearly deer count from now on.
  - B. The warden will avoid the risk of having too many deer on the reservation.
  - C. The warden will gain experience in building population growth models.
  - D. The warden will gain knowledge about past deer herd sizes.
  
3. Computer models which are used to predict weather often have limited accuracy because
  - A. weathermen mistrust the computer.
  - B. the data and assumptions used to build the model may contain errors.
  - C. it takes too much time to compute predictions.
  - D. weather cannot be predicted using models.

4. Directions: On the blank in front of each definition, place the letter of the term that best matches the definition.

<u>Definition</u>	<u>Term</u>
___ 1) A statement which is accepted as true without proof.	A. Assumption
___ 2) The process of making corrections to a computer model.	B. Data
___ 3) Numbers used in making a computer model.	C. Equation
___ 4) A letter or word that stands for a number.	D. Pattern
___ 5) A relation between two or more things.	E. Hypothesis
	F. Revision
	G. Variable

5. Listed below are the steps involved in the modeling process. Put the steps in the correct order.

- |   |     |
|---|-----|
| A. Build the model using variables and assumptions.               | ___ |
| B. Make observations and measurements.                            | ___ |
| C. Revise the model if necessary.                                 | ___ |
| D. Find a pattern in the measurements.                            | ___ |
| E. Check your measurements with the figures that the model gives. | ___ |

6. Mr. James, a city planner, assumed that the population growth rate of Allstate City is 1.7. He based this assumption on the following population figures.

<u>Year</u>	<u>Population (Thousands)</u>
1971	20
1972	30
1973	45
1974	68
1975	102

Mr. James built a population growth model using his assumed growth rate of 1.7 and the model produced the following figures.

<u>Year</u>	<u>Population (Thousands)</u>
1971	20
1972	34
1973	58
1974	99
1975	168

What should Mr. James do next?

- A. He should go on to look at other city problems.
- B. He should apply this model to Baxter City.
- C. He should revise his model.
- D. None of the above.

7. The data below represent the population size of a country for the years 1965 through 1969.

<u>Year</u>	<u>Population</u> (Thousands)
1965	12
1966	36
1967	108
1968	324
1969	972

Which of the following statements best describes the way the population is growing?

- A. The population is getting smaller.
  - B. The population is doubling every year.
  - C. The population is tripling every year.
  - D. The population is staying the same in size.
8. In the space below, write a computer program which will give the population figures shown in question #7.

9. For the population data given below, what is the rate of growth of the population?

<u>Year</u>	<u>Population (Thousands)</u>
1960	14
1961	21
1962	31
1963	47
1964	71

- A. 1.5
- B. 0.67
- C. 0.5
- D. 3.0

10. In 1970, the world's supply of aluminum was estimated to be 1200 million tons. In the year 1970, the world's nations mined 12 million tons.

Write a computer program that will print out the amount of aluminum left at the end of each year up to the year 1980. Your computer printout should look similar to the figures shown below.

<u>Year</u>	<u>Amount of Aluminum Left</u> (Million Tons)
1970	1200
1971	1188
1972	1176
1973	1164
1974	1152
1975	1140
1976	1128
1977	1116
1978	1104
1979	1092
1980	1080

11. What assumption was made in building the model in question #10?
- The aluminum supply will increase in years to come.
  - The aluminum supply will remain the same in years to come.
  - The number of tons of aluminum mined each year stays the same.
  - The number of tons of aluminum mined each year is doubled.
12. What final outcome is predicted by the model in question #10?
- The world's aluminum supply will grow in the future.
  - The world's aluminum supply will stay the same if we mine 12 million tons per year.
  - The world's aluminum supply will stay the same if we mine less than 12 million tons per year.
  - The world's aluminum supply will someday run out.
13. The figures below show the crackerjack production and the population size for the country of Transylvania for the years 1969 through 1973.

<u>Year</u>	<u>Crackerjack Production</u> (Tons)	<u>Population</u> (Thousands)
1969	60	9
1970	72	10
1971	86	11
1972	103	12
1973	124	13

Mr. Werewolf, the president of Transylvania, knows that the production of crackerjacks is increasing and that the population is also increasing. He wants to find out if his people are still getting the same amount (or more, or less) of crackerjacks to eat each year. He decides to build a computer model using the figures above. The best way to do this would be

- divide the crackerjack production by the population size for each year and look at the resulting figures.
- look to see if the crackerjack production figures have doubled.
- find out if the population figures have remained the same.
- graph the population figures and the crackerjack production figures on two separate charts and compare the graphs.



14. The figures below show the Gross National Product (GNP) and the Gross National Product per person for a small developing country. The Gross National Product is the value in dollars of all new goods and services produced during a particular year.

<u>Year</u>	<u>GNP (Millions)</u>	<u>GNP Per Person</u>
1971	\$2000	\$197
1972	\$2350	\$200
1973	\$2500	\$199
1974	\$2800	\$201
1975	\$3000	\$199

Using the above data, which of the following is not a correct conclusion.

- A. The country as a whole is getting richer.
  - B. The wealth of individual people has stayed the same.
  - C. The population is increasing.
  - D. Each person is getting richer.
15. Mr. Brown, a Connecticut farmer, owns a herd of cows. What is the growth rate of his cow herd from the following information?

<u>Year</u>	<u>Herd Size (Number of Cows)</u>
1970	10
1971	23
1972	53
1973	122
1974	281
1975	646

- A. 23
  - B. 13
  - C. 0.43
  - D. 2.3
16. Old MacDonald planted corn on 5 acres of his farm. Each acre gave him 25 bushels of corn. The total amount of corn was found by multiplying 25 bushels by the number of acres he planted (5).

The following program finds the total amount of corn in line 20 and prints it out in line 30. (T = total amount of corn, A = number of acres).

```

10 LET A = 5
20 LET T = 25*A
30 PRINT T
40 END

```

If Old MacDonald decided to plant corn on 15 acres of his farm instead of 5 acres, he would do which of the following:

- A. Change line 10 to: LET A = 15.
- B. Change line 20 to: LET T = 15\*A.
- C. Change line 20 to: LET T = 3\*A.
- D. Change line 30 to: PRINT A.

17. Farmer Jones also plants corn, but gets 32 bushels of corn from each acre. He wants to use the same program as Old MacDonald but needs to do which of the following:

- A. Change line 10 to: LET A = 32.
- B. Change line 20 to: LET T = 32\*A.
- C. Change line 20 to: LET T = 7\*A
- D. Change line 30 to: PRINT 32.

18. The governor of Wetstate assumed that the yearly rainfall growth rate was 2. He based this assumption on the following numbers:

<u>Year</u>	<u>Amount of Rainfall (Inches)</u>
1965	2
1966	3.9
1967	7.8
1968	15.2
1969	30.1

The governor built a model using his assumed rainfall growth rate of 2 and got the following numbers from the model:

<u>Year</u>	<u>Amount of Rainfall (Inches)</u>
1965	2
1966	4
1967	8
1968	16
1969	32

Based on the above information, the governor of Wetstate should change his rainfall growth rate to:

- A. a number only slightly smaller than 2.
- B. a number only slightly bigger than 2.
- C. a number very much smaller than 2.
- D. a number very much bigger than 2.

19. In a small midwest town, there are 1000 people and 2000 automobiles. In the computer programs below, the number of people is P and the number of autos is A. The number of autos per person is X. Fill in the blank in line 30 of the program so that the program will calculate the number of autos per person (X).

```

10 LET P = 1000
20 LET A = 2000
30 LET X = _____
40 PRINT X
50 END

```

20. The air pollution from carbon dioxide gas in a city amounts to 2 cubic feet each week. Suppose the wind does not blow away this pollution for 30 weeks. To find how much carbon dioxide is in the air after 30 weeks, which of the following would you use?

- A.  $\frac{2}{30}$
- B.  $30 \div 2$
- C.  $30 - 2$
- D.  $2 \times 30$

\*\*\*\*\*

PLEASE ANSWER THE FOLLOWING ALSO: .

1. Was the test

- A. too hard.
- B. so-so.
- C. too easy.

2. Was the test

- A. easy to read.
- B. hard to read.
- C. so-so to read.

3. If you answered B or C to question #2, list the numbers of the questions that gave you difficulty in reading. \_\_\_\_\_

4. Any other comments, ideas or questions you can make about the test would be appreciated. Use the space below.

5. Have you ever had any computer programming?

## APPENDIX D

## MODELING CURRICULUM MATERIALS

COMPUTER MODELING FOR JUNIOR HIGH SCHOOL STUDENTS

A Handbook for Teachers

Anne Pasquino

Cynthia Walowicz



## Modeling Curriculum Materials Outline

- I. Exponential Growth Models -- UNIT ONE
  - A. Introduction to Modeling
  - B. Population Growth (Sample Model)
  - C. Economic Growth
    - 1. GNP
    - 2. GNP/Capita
- II. Food Production and Nonrenewable Resources -- UNIT TWO
  - A. Food Production
    - 1. Arable Land Usage (Sample Model)
    - 2. Fresh Water Usage
  - B. Food Production/Capita
  - C. Nonrenewable Natural Resources
    - 1. Chromium Reserves (Sample Model)
    - 2. Energy Consumption
- III. The Technological Optimist -- UNIT THREE
  - A. Technology -- Exponential Growth?
  - B. Effect of Technology on Food Supply
  - C. Effect of Technology on Population

Several growth models which appear in the curriculum materials were inspired by and in some cases directly adapted from those which appear in The Limits to Growth by Meadows, et al., Universe Books, 1972.

## EXPONENTIAL GROWTH MODELS

## UNIT ONE

- A. Introduction to Modeling
- B. Population Growth
- C. Economic Growth
  - 1. GNP
  - 2. GNP/Capita

Note to the Teacher:

In this unit the student is introduced to the concept of computer modeling and is taught to construct a simple computer program which models exponential growth.

The student is shown data which depicts a quantity which grows exponentially with time and is asked to find an average growth rate. Using the average growth rate, the student writes a program which will predict the size of the quantity in future years.

## A. INTRODUCTION TO MODELING

### "What Is A Model?"

The first concept we wish to present is that of a "model." You might begin by asking the students what they think is meant by the term "model" or what it means when someone "builds a model of something." Some possible responses might include the type of physical models children are most familiar with, e.g., model airplanes, model cars, model railroads, etc. Use these responses as a springboard to introduce and discuss other types of models used for testing purposes, such as wind tunnels, the zero-gravity chamber used by astronauts, the Link trainer, etc. Do not worry about making a distinction between simulations and models. From physical models move to conceptual models or scientific theory models. Try to use examples of scientific models the students are familiar with from their science course. For example, models of the atom or models of the universe might be discussed. Finally, mathematical models should be mentioned and at least one example given. Mathematical models which involve simple equations, such as Newton's Second Law of Motion ( $F = ma$ ) or Ohm's Law ( $E = IR$ ), can be used.

Some additional examples of models might include:

#### Physical Models

- Wind tunnel
- Link trainer
- Zero-gravity chamber of astronauts
- Road maps and maps in general
- Planetarium
- Fishtank
- Terrarium
- Model airplanes, cars, railroads, ships
- World globe
- Dioramas and architectural models
- Various games such as MONOPOLY, chess, etc.
- Erector set constructions

### Conceptual Models (Ideas and theories about how things work)

Kinetic theory of gases  
 Freud's theories  
 Atomic theory  
 Models of the universe (Expanding Universe, etc.)  
 Magnetic theory  
 Electric theory

### Mathematical Models

Work = Force x Distance  
 Velocity = Acceleration x Time  
 Distance = Velocity x Time  
 Density = Weight/Volume  
 Pressure = Force/Area  
 Mechanical Advantage = Resistance Force/Effort Force  
 Newton's Law of Universal Gravitation  $F =$   

$$G \frac{m_1 m_2}{s^2}$$

Do not discuss any one model in detail at this point. It is sufficient to acquaint the students with the three types of models: physical, conceptual and mathematical. Do not try to pigeon-hole the models as belonging to one of the three categories mentioned since no one model is exclusively physical or conceptual or mathematical.

After presenting and discussing several kinds of models, ask the students to try to pick out some common characteristics of the models which might lead to a definition of "model". List the characteristics which the students suggest on the blackboard.

Possible characteristics:

models describe something or represent something

models are like the thing they represent in certain aspects

models are in some ways different from the thing they represent

models are always man-made, whereas the thing they represent may or may not be man-made

models are usually simpler than the thing they describe

models are incomplete

models change over time (are revised, refined and updated as time passes)

models are never "perfect"

measurements or observations are used to develop a model

models can take different forms or can be expressed in various ways (e.g., physical, conceptual, mathematical)

It may be extremely optimistic to think that the characteristics listed above will be extracted from the students. Perhaps considerable prodding and leading will be necessary to get even a few characteristics. Don't be concerned with duplicating the above list exactly. This is only a list of possible characteristics. There may be other characteristics which the students will come up with. Some responses may involve value judgements such as "a model is helpful," "models are good," etc.

The important understandings which the student should come away with at this point are:

- (1) models describe or represent something.
- (2) models can take many forms--physical, conceptual, mathematical.

A suitable definition of a model at this stage would be:

A model is a description of something in the 'real world.'  
The description can take the form of a physical replica,  
an idea or theory or a mathematical equation.

Some students may also come away with the following notions concerning models:

models are simplifications

models are incomplete

models are subject to change



## "Why Use Models?"

After defining a model, the examples of various models should be reexamined in an attempt to discover why people build models. Some of the advantages and uses of models may have already come out in the previous discussion. List the advantages on the blackboard.

### Advantages:

Models are economical -- It is cheaper to build and experiment with a model than to experiment with the real thing. Example: Airplanes are tested in a wind tunnel before a full-scale plane is built.

Models are observable -- Often it is impossible to carry out an experiment in the real world because scientific instruments do not exist or have not been developed to the point where direct observation is possible. For example: Direct viewing of sub-atomic particles. Also, there are times when direct viewing may interfere with the very process we are trying to observe.

Models are controllable -- The parameters of a model can be varied (e.g., birth rate of a population) and their effects studied. Time can be speeded up or slowed down--allowing us to look at the future or notice effects that would otherwise go unobserved.

Models are reproducible -- Results of physical phenomenon can be simulated and their causes studied and understood. For example: Geology models aid in understanding earthquakes and other irreversible disasters. These phenomena can be reproduced and studied over and over again with a model.

Models are changeable -- Underlying assumptions, variables and even the entire design of a model can be changed to make improvements and/or study their effects. It is simpler and less expensive to redesign a model airplane than it is to alter its full-scale counterpart.

Models are safe --

It is safer to discover and prepare for the hazards involved in a task via a model. For example: The astronauts simulated many flights with their equipment in a benign laboratory environment before actually venturing into space. (Population control models which affect animal species prevent man from having an irreversible detrimental effect on real animal populations. Detroit car manufacturers run many tests using a "dummy" or model of a man in testing their automobiles for safety.)

Models are useful for prediction and planning -- The model presents a means of examining trend patterns, forecasting outcomes and making decisions about what course to follow in the future.

Models are an aid to communication and thought -- A model helps to clarify and organize thoughts when dealing with a complex or abstract idea.

Models are an aid to teaching and learning -- Link trainers were used extensively and successfully in the second world war to train pilots. Computer simulations are being used as a substitute for expensive laboratory equipment to help train biologists, chemists and physicists.

The disadvantages of using models should be touched on briefly at this point. It should be made clear that models do not represent reality exactly and accurately. A model is a simplification of the real world and predictions of the model should be constantly checked to see how well they conform with reality. We will return to this point later when the students begin constructing models of their own.

## B. BUILDING A SIMPLE POPULATION GROWTH MODEL

Consider the figures below which represent the population size (in millions) for the United States for the years 1900 to 1970. (You could have the students look up these figures in an almanac and do the rounding off.)

<u>Year</u>	<u>Population (millions)</u>
1900	76
1910	92
1920	106
1930	123
1940	132
1950	151
1960	179
1970	203

Source: Information Please Almanac, 1965.  
Reader's Digest Almanac, 1976.

### A First Attempt

Suppose we wish to predict the population size in 1980. We could try to build a model of population growth by doing the following:

- (1) Find the increase in population size for each 10 years.
- (2) Find the average increase.
- (3) Write a computer program which increases the population size by this amount every 10 years.

Example:

<u>Year</u>	<u>Population (millions)</u>
1900	76
1910	92
1920	106
1930	123
1940	132
1950	151
1960	179
1970	203

The average increase = 18 million people in 10 years.

Computer Program:

```

10 LET Y = 1900
20 LET P = 76
30 PRINT Y,P
40 LET Y = Y+10
50 LET P = P+18
60 GO TO 30
70 END

```

Note: The program contains an endless loop  
and must be terminated manually.

Now let's run the program and see how the population figures from  
our model compare with the known population values for 1900-1970.

Printout:

<u>Year</u>	<u>Predicted</u>	<u>Actual</u>	<u>Predicted-Actual</u>
1900	76	76	0
1910	94	92	+2
1920	112	106	+6
1930	130	123	+7
1940	148	132	+16
1950	166	151	+15
1960	184	179	+5
1970	202	203	-1

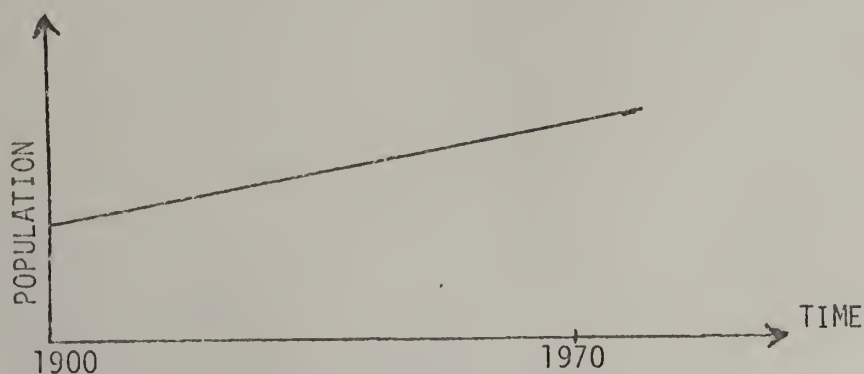
Discuss the "goodness of fit" for this model and also find out what the model predicts for 1980.

You might also point out the following to students:

The model we have just built assumes a constant increase in the size of the population each year. That is,

$$\text{POPULATION (NEW)} = \text{POPULATION (OLD)} + \text{FIXED INCREASE}$$

If we graph population size against time (in years) we get a straight line. Hence, our model is called a linear model, or population is said to increase linearly with time in this model.



Discuss the need for revising the model. Also discuss the possibility that maybe the "increase" is not a good way to describe the growth since the increase is far from constant in size.

### A Second Try

Suppose we construct a model by taking the ratio of two successive years.

<u>Year</u>	<u>Population</u> (millions)	
1900	76	
1910	92	$92/76 = 1.21$
1920	106	$106/92 = 1.15$
1930	123	$123/106 = 1.16$
1940	132	$132/123 = 1.07$
1950	151	$151/132 = 1.14$
1960	179	$179/151 = 1.18$
1970	203	$203/179 = 1.13$

Explain to the students that this ratio is the growth rate of the population. (Instead of a flat increase, we are looking at a percentage increase -- 92 represents a 21% increase in 76 -- or 92 is 121% of 76.)

Average growth rate = 1.15 or 115%.

Program:

```

10 LET Y = 1900
20 LET P = 76
30 PRINT Y,P
40 LET Y = Y+10
50 LET P = 1.15*P
60 GO TO 30
70 END

```

Printout:

<u>Year</u>	<u>Predicted</u>	<u>Actual</u>	<u>Predicted-Actual</u>
1900	76	76	0
1910	87	92	-5
1920	100.5	106	-5
1930	116	123	-7
1940	133	132	+1
1950	153	151	+2
1960	176	179	-3
1970	202	203	-1



Discuss the "goodness of fit" of this model as compared to the first model.

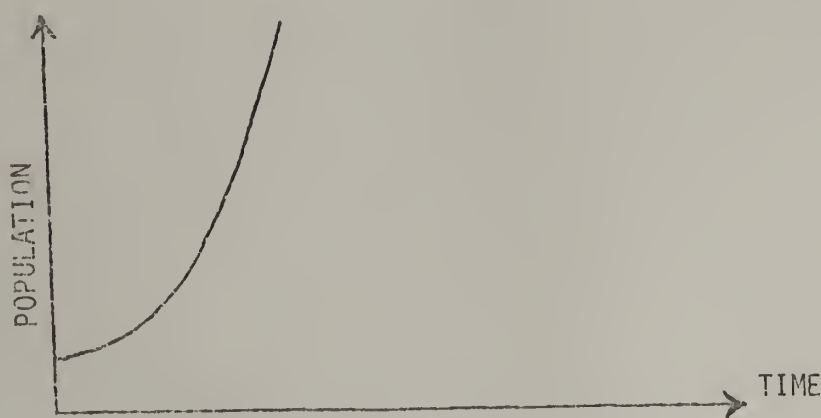
This would be a good point at which to discuss the fact that no model is perfect and we judge the "goodness" of a model by how well it reflects the real world. Discuss the need for revision of models based on new data, etc.

At this point, the students might experiment with the model. Ask the students what they think would happen if the growth rate were equal to one. Rewrite the program using a growth rate of one to verify conjectures. Try the model using growth rates of less than one also (see exercises).

The model we have just built assumes that the population this year is a constant multiplied by last year's population.

$$\text{POPULATION (NEW)} = \text{CONSTANT (GROWTH RATE)} \times \text{POPULATION (OLD)}$$

If we graph population against time (for a growth rate  $> 1$ ), we get a curve which rises more steeply than a straight line and has the following shape. Such a curve is known as an exponential curve. We say that population rises exponentially with time.



It would be nice to show the students a computer generated graph of this exponential growth curve. The students need not write the graph program nor be expected to understand it. However, they should be capable of running it. The graph program is shown on the following page along with several printouts, each using a different scale. (Note: From time to time the teacher will have to make modifications in the graph program for other applications.)

```
5 PRINT "WHAT IS YOUR INITIAL POPULATION (IN MILLIONS)";
10 INPUT P
15 PRINT "WHAT IS YOUR SCALE";
20 INPUT S
22 PRINT "ONE ASTERISK REPRESENTS" S "MILLION PEOPLE"
25 PRINT "WHAT IS YOUR GROWTH RATE";
30 INPUT R
40 LET M=INT(P/S + .5)
50 IF M>72 THEN 999
60 FOR I=1 TO M
70 PRINT "*";
80 NEXT I
90 PRINT
100 LET P=R*P
110 GO TO 40
999 PRINT "LINE LIMIT EXCEEDED"
1000 END
```

RUN

WHAT IS YOUR INITIAL POPULATION (IN MILLIONS)? 76

WHAT IS YOUR SCALE? 75

ONE ASTERISK REPRESENTS 75 MILLION PEOPLE

WHAT IS YOUR GROWTH RATE? 1.15

\*

\*

\*

\*\*

\*\*

\*\*

\*\*

\*\*\*

\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

LINE LIMIT EXCEEDED

RUN

WHAT IS YOUR INITIAL POPULATION (IN MILLIONS)? 76

WHAT IS YOUR SCALE? 25

ONE ASTERISK REPRESENTS 25 MILLION PEOPLE

WHAT IS YOUR GROWTH RATE? 1.15

\*\*\*

\*\*\*

\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

LINE LIMIT EXCEEDED

RUN

WHAT IS YOUR INITIAL POPULATION (IN MILLIONS)? 76

WHAT IS YOUR SCALE? 50

ONE ASTERISK REPRESENTS 50 MILLION PEOPLE

WHAT IS YOUR GROWTH RATE? 1,+.15

\*\*

\*\*

\*\*

\*\*

\*\*\*

\*\*\*

\*\*\*\*

\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

\*\*\*\*\*

LINE LIMIT EXCEEDED

At this point, the students should be asked to build population growth models for Europe, Asia, Africa and the world (see exercises).

### Exercises

The computer program we have just written is a model of the population growth observed. Let us experiment with the model and see what we can learn.

- (1) Vary the initial population size by editing line 20.

EXAMPLE: 20 LET P = 25    or    20 LET P = 100

Since it is tedious to repeatedly edit line 20, this would be a good point at which to introduce the INPUT statement if the students are not already familiar with it.

- (2) Vary the rate at which the population is growing.

We were able to see from the data that the population size increased by 15% each year. This means that for each person in the population this year, there will be 1.15 people next year. If the notion of 1.15 people seems strange to the students, then state the condition as: For every 100 people this year, there will be 115 next year. The constant 1.15 in line 50 of our program is the rate of growth of the population.

Change the growth rate to 10, 20, 50, 100, etc., and run the program to see what effect a larger growth rate will have on the growth of the population.

It may be convenient to modify the program to allow the value of R to be INPUT (R is the growth rate).

- (a) Change the growth rate to 1 and run the program. What effect does a growth rate of 1 have on the population growth?
- (b) Change the growth rate to 0.5, 0.25, 0.125, etc., and run the program. What effect do growth rates less than 1 have on population growth? How would you interpret a growth rate of 0.5 or  $\frac{1}{2}$ ? (Answer: For every two people in the population this year, there will be one person next year.)



You might want to again point out to students that the rate of growth is a ratio.

$$\text{Rate of Growth} = \frac{\text{Population in year } (x+1)}{\text{Population in year } x}$$

- (3) Using the figures in Table 1, construct population growth models for the regions of the world which are listed. Have the students supplement and table by consulting recent almanacs for the 1975 and 1976 figures.
- (4) Using the table, construct a population growth model for the world. At what rate is the world's population increasing?

TABLE 1  
POPULATION FIGURES FOR REGIONS OF THE WORLD  
(IN MILLIONS)

Year	Africa	North America	Latin America	Asia	Europe	U.S.S.R.	Oceania	World
1970	346	226	283	2027	459	243	19.3	3610
1971	354	228	291	2104	466	245	19.8	3706
1972	364	230	298	2154	469	248	20.2	3782
1973	381	233	307	2160	467	250	20.3	3818
1974	391	235	315	2206	470	252	20.9	3890
1975								
1976	413			2353				4019

Sources: United Nations Statistical Yearbook, 1975; World Almanac, 1977.

## Reflection

Now that we have had some experience in model building, let's think about what steps were involved in building the model.

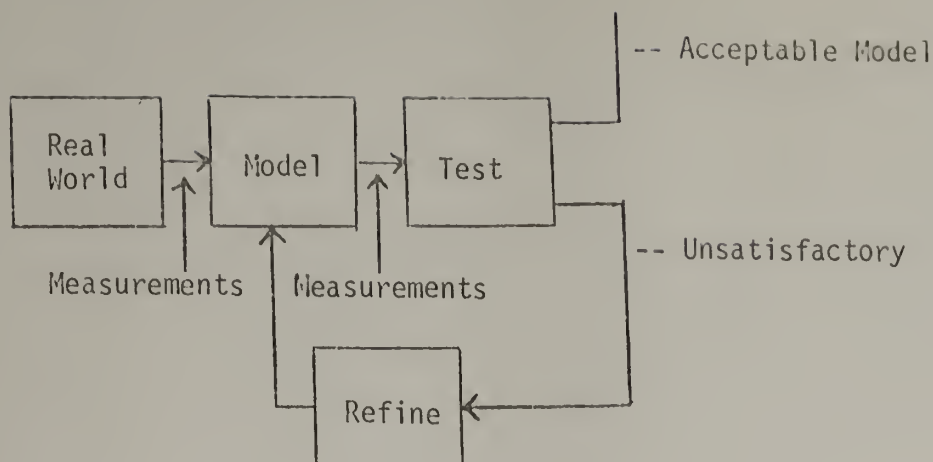
First, we looked at some data which had been observed and recorded.

Second, we studied the data and were able to see a pattern or relation between the population size for one year and the following year. Sometimes the pattern was not clear and we were not sure of ourselves. We simplified the data sometimes to help us see a pattern. Simplifying the data might include rounding off numbers and/or taking averages.

Third, we built our model (the computer program) based on the precise way we thought the population was growing. We included in the program things like "initial population" and "growth rate". These quantities were included because they were needed to describe the population growth. The values of these quantities change as we ran the program. Things with such changing values are called variables.

Finally, we compared the output of our model with the original data to see if our model was "good". If our computer model did not give us values which were close to the values in the original data, then we revised the model. (The model should also be checked against additional data if available.) Hence, the steps in the modeling process are as follows:

- (1) Make observations and measurements (collect data).
- (2) Look for a relation or pattern in the measurements. Simplify the data if necessary.
- (3) Build the model using what you think are the important variables and assumptions.
- (4) Test the model by checking the predictions of the model against your data.
- (5) If the predictions do not check, then revise the model.



(Figure taken from The Man-Made World, McGraw-Hill; page 177.)

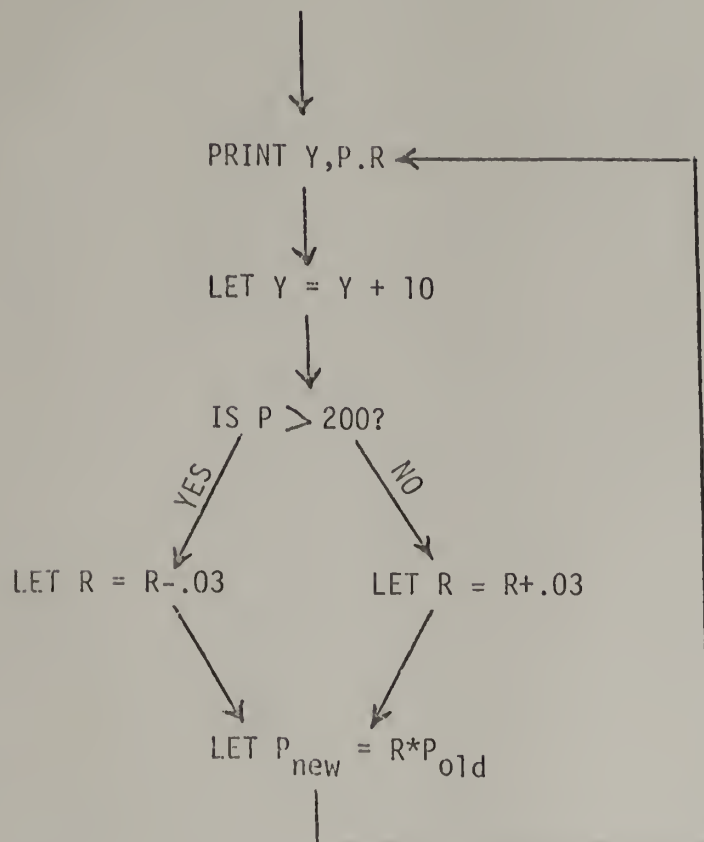
#### Experimentation With Population Growth Models (Optional)

Suppose that starting in 1970 we want to stabilize the population of the United States at 200 million people. That is, we wish to keep the population of the country around 200 million people.

Since the 1970 population is already 203 million people, we cannot continue to grow at the present rate (the rate used in the model which is 1.15) and expect the population to stay near 200 million. Therefore, we decide to decrease the growth rate by .03 every ten years if the population exceeds 200 million and increase the growth rate by .03 every ten years if the population is less than 200 million.

Consider the following model:

Initialize:  $P = 203$  million  
 $Y = 1970$   
 $R = 1.15$



<u>LIST</u>	<u>RUN YEAR</u>	<u>POPULATION</u>	<u>GROWTH RATE</u>
10 LET Y = 1970	1970	203	1.15
20 LET P = 203	1980	227.36	1.12
25 LET R = 1.15	1990	247.8224	1.09
27 PRINT "YEAR", "POPULATION",	2000	262.6918	1.06
"GROWTH RATE"	2010	270.5725	1.03
30 PRINT Y,P,R	2020	270.5726	1
40 LET Y = Y+10	2030	262.4554	.9700001
50 IF P > 200 THEN 100	2040	246.7081	.9400001
60 LET R = R+.03	2050	224.5044	.9100001
70 GO TO 200	2060	197.5639	.8800001
100 LET R = R-.03	2070	179.7832	.9100001
200 LET P = R*P	2080	168.9962	.9400001
210 GO TO 30	2090	163.9263	.9700001
999 END	2100	163.9263	1
	2110	168.8441	1.03
	2120	178.9748	1.06
	2130	195.0825	1.09
	2140	218.4925	1.12
	2150	238.1568	1.09
	2160	252.4462	1.06
	2170	260.0196	1.03
	2180	260.0196	1
	2190	252.2191	.9700001
	2200	237.086	.9400001
	2210	215.7483	.9100001
	2220	189.8585	.8800001
	2230	172.7713	.9100001
	2240	162.405	.9400001
	2250	157.5329	.9700001
	2260	157.5329	1
	2270	162.2589	1.03
	2280	171.9944	1.06
	2290	187.4739	1.09
	2300	209.9708	1.12
	2310	228.8682	1.09
	2320	242.6003	1.06
	2330	249.8783	1.03
	2340	249.8784	1
	2350	242.3821	.9700001
	2360	227.8392	.9400001
	2370	207.3337	.9100001
	2380	182.4537	.8800001
	2390	166.0329	.9100001
	2400	156.0709	.9400001
	2410	151.3888	.9700001
	2420	151.3888	1
	2430	155.9305	1.03
	2440	165.2863	1.06
	2450	180.1621	1.09
	2460	201.781	
	STOP		



The output indicates that the population is oscillating around 200 million and gradually getting closer to 200 million on the average. The population is taking a long time to stabilize. This can be seen more clearly by generating a graph of the population (next page) against time. You might have the students investigate the effects of different rate increments (0.05, 0.10, etc.) or have them shorten the time span between rate adjustments.

This particular model could serve to open up several topics for discussion. In particular, the effect of negative and positive feedback loops, and the time delay involved between the implementation of a change (change in growth rate) and the response to that change (drop or rise in population size).



### C. ECONOMIC GROWTH

#### 1. GNP

A second quantity that has been increasing in the world even faster than human population is industrial output. The industrial output of a country is measured by its Gross National Product or GNP. Very simply, the GNP is a measure of the total amount of new goods and services produced during a given period. To get the students started on an economic growth model, furnish them with the figures below for the United States.

<u>Year</u>	<u>GNP (billions)*</u>
1960	\$506.0
1961	523.3
1962	563.8
1963	594.7
1964	635.7
1965	688.1
1966	753.0
1967	796.3
1968	868.5
1969	935.5
1970	982.4
1971	1063.4
1972	1171.1
1973	1306.3
1974	1406.9
1975	1498.8 (preliminary figure)

(\*Current dollars used, not constant dollars.)

Source: The Mathematics Teacher, January 1977.

Work with the students and have them construct an economic growth model by following the same procedure used in constructing the second population growth model. They should find that the average growth rate for the United States GNP is 1.075.

Have the students do the exercise which asks them to build a GNP model for selected countries (India, Sweden, and Ghana). Also have them construct a world GNP growth model.

## 2. GNP/Capita

For the world model, industrial output is growing at about 7% per year. Since world population is growing at about 2% per year (students should have this figure from the world population model), it might at first appear that the material standard of living of the world's people is rising. This is a good discussion point. The fallacy here is the assumption that the world's growing industrial output is evenly distributed among the world's citizens. This leads nicely into per capita economic growth rates. Take a few selected countries to illustrate this point. Use the same countries for which you have already built a GNP model (see exercise 2). The students should be able to see that most of the world's industrial growth is actually taking place in the already industrialized countries, where the rate of population growth is comparatively low. Use the graphing program to print out GNP and GNP/capita for a few developed and undeveloped countries.

### Exercises

- (1) Given the following figures which represent the GNP for the countries listed, construct a computer model of the growth of GNP for each country.

Year	Ghana	India (GNP in billions of dollars)	Sweden	U.S.
1970	2.458	52.9	32.6	982
1971	2.067	57.8	38.0	1063
1972	2.151	58.2	43.6	1171
1973	2.857	61.1	50.1	1306
1974	3.057	64.2	56.1	1407

- (2) Using the GNP figures given above and the population figures given below, construct a computer model of the GNP/capita for each country.

Year	Ghana	India (millions of people)	Sweden	U.S.
1970	8.63	538.88	8.04	204.88
1971	8.86	550.82	8.10	207.05
1972	9.09	562.99	8.12	208.84
1973	9.36	574.22	8.14	210.40
1974	9.61	586.06	8.16	211.91

- (3) Using the GNP figures for the world which are given below, and the population figures for the world given earlier, construct a computer model of GNP/capita for the world.

Year	World GNP (billion)	Population (million)
1970	3219.3	3610
1971	3380.3	3706
1972	3573.4	3782
1973	3831.0	3818
1974	3959.7	3890

## FOOD PRODUCTION AND NONRENEWABLE RESOURCES

## UNIT TWO

## A. Food Production

1. Arable Land Usage
2. Fresh Water Usage

## B. Food Production/Capita

## C. Nonrenewable Resources

1. Chromium Reserves
2. Energy Consumption



Note to the Teacher:

In this unit the student builds models in which the rate of consumption of a quantity or resource is known and the quantity is being consumed linearly or exponentially. The computer models are used to predict the amount of the quantity consumed each year. Since the quantities studied are finite, the computer model is used to estimate the time at which the quantity or resource will be exhausted.

## A. FOOD PRODUCTION

### 1. Arable Land Usage

We have been studying population models and have found that the population of the world is increasing exponentially. As more and more people inhabit the earth, more food is necessary to feed them. Hence, the world must produce more food each year. The primary resource for producing food is land. It has been estimated that the average amount of land necessary to produce enough food to feed one person for a year is 0.4 hectares (one hectare = 2.471 acres or one hectare is approximately 2.5 acres).

Can we build a computer model which tells us how much land is necessary to feed the world's population each year?

$$\begin{array}{l} \text{Number of Hectares} \\ \text{of Land Needed} \end{array} = \left( \frac{0.4 \text{ Hectares}}{\text{Person}} \right) \times \left( \begin{array}{l} \text{Number of} \\ \text{People} \end{array} \right)$$

The computer statement that would be needed to express this is:

$$\text{LET H} = 0.4 * \text{P}$$

where H = number of hectares of land needed and  
P = number of people or population size.

Have the computer print out the land needed for each year from 1900 to 2500.

Alter the graphing program to plot a graph of hectares of land vs. time in years.

### Discussion and Exercises:

- (1) Compare the graph of hectares of land needed with the graph for population growth from the last unit. What do you notice about their shapes? Are the models similar?

	<u>RUN YEAR</u>	<u>POPULATION (MILLIONS)</u>	<u>HECTARES NEEDED (MILLIONS)</u>
10 LET P = 3610	1970	3610	1444
20 LET R = 1.019	1980	3678.59	1471.436
30 LET Y = 1970	1990	3748.483	1499.393
40 PRINT "YEAR",	2000	3819.704	1527.882
"POPULATION",	2010	3892.279	1556.912
"HECTARES NEEDED"	2020	3966.232	1586.493
45 PRINT " ", "(MILLIONS)",	2030	4041.59	1616.636
"(MILLIONS)"	2040	4118.381	1647.352
50 LET H = 0.4*P	2050	4196.63	1678.652
60 PRINT Y,P,H	2060	4276.366	1710.546
70 LET P = R*P	2070	4357.617	1743.047
80 LET Y = Y+10	2080	4440.411	1776.165
90 GO TO 50	2090	4524.779	1809.912
100 END	2100	4610.75	1844.3
	2110	4698.354	1879.342
	2120	4787.623	1915.049
	2130	4878.588	1951.435
	2140	4971.281	1988.512
	2150	5065.735	2026.294
	2160	5161.984	2064.794
	2170	5260.062	2104.025
	2180	5360.003	2144.001
	2190	5461.843	2184.737
	2200	5565.618	2226.247
	2210	5671.364	2268.546
	2220	5779.12	2311.648
	2230	5888.924	2355.569
	2240	6000.813	2400.325
	2250	6114.828	2445.931
	2260	6231.01	2492.404
	2270	6349.399	2539.76
	2280	6470.038	2588.015
	2290	6592.969	2637.187
	2300	6718.235	2687.294
	2310	6845.881	2738.353
	2320	6975.953	2790.381
	2330	7108.496	2843.398
	2340	7243.558	2897.423
	2350	7381.185	2952.474
	2360	7521.428	3008.571
	2370	7664.335	3065.734
	2380	7809.957	3123.983
	2390	7958.346	3183.338
	2400	8109.555	3243.822
	2410	8263.636	3305.455
	2420	8420.	
	STOP		

If more and more land is needed each year for food, where will this land come from? Will we run out of land?

After the above questions have been discussed, you might present the students with the following information:

- (2) It has been estimated that there are approximately 3.2 billion hectares of land suitable for agriculture on the earth (7.86 billion acres). Roughly half of this land, the richest most accessible half, is under cultivation today. The remaining land will require a large amount of money to reach, clear, irrigate or fertilize before it is ready to produce food (Limits to Growth, p. 48). Assuming the world's people would be willing to pay the cost of cultivating this land so that all 3.2 billion hectares are used to produce food, how many people could the earth theoretically feed?

In what year would we be using all the land available?

What would happen in the years that follow?

Why are people worried about population growth?

The students should be able to answer the question of land usage by looking at the computer program which they have written. After the students have discussed the results of their program, you might suggest the following activities:

- (3) Could something happen which might enable the earth to feed more people without decreasing the population? Could the earth become more productive? If so, how?

Suppose the land planted to raise food were twice as productive as it is now, then how many hectares would be necessary to feed one person for a year (0.2 hectares/person)?

Alter your model using this assumption. Run the program. What difference would this assumption make (in the short run; in the long run)?

- (4) What if the land planted were four times as productive? Alter the model again. What difference would this make to the world in the long run? (0.1 hectares/person is needed.)

Here we have discussed only one possible limit to food production--arable land. There are other possible limits--for example, second in importance only to land is the availability of fresh water. There is an upper limit to the fresh water runoff from the land areas of the earth each year and there is also an exponentially increasing demand for that water. We could build a model similar to that for arable land which would show the increasing demand curve for water and the constant average supply.

## 2. Fresh Water Usage

It has been estimated that the rate of fresh water usage in the United States is presently 16,000 gallons/person each day. This does not mean that each person uses 16,000 gallons to cook and wash with each day. This rate of 16,000 gallons/person takes into account the fact that water is used to generate electricity, manufacture goods and grow food, used in transportation and recreation, etc. By taking the total number of gallons used daily for all these purposes as well as that used for drinking, cooking, washing and sewage and dividing by the population size, we obtain the consumption rate of 16,000 gallons/person for each day.

The average total precipitation in the United States is 5,000 billion gallons per day. Have the students write a computer program that will answer the following:

Based on the fresh water usage rate of 16,000 gallons/person each day, how large would the population be before we are using all of the daily precipitation of 5,000 billion gallons/day.



```

10 LET P = 208E+6
20 LET Y = 1974
25 PRINT "YEAR", "POPULATION", "GALLONS/DAY"
30 LET C = 160000*P
40 PRINT Y,P,C
45 LET Y = Y+10
50 IF C 50000E+9 THEN 999
60 LET P = 1.15*P
70 GO TO 30
999 END

```

READY

<u>RUN</u> <u>YEAR</u>	<u>POPULATION</u>	<u>GALLONS/DAY</u>
1974	2.080000E+8	3.328000E+12
1984	2.392000E+8	3.827200E+12
1994	2.750800E+8	4.401280E+12
2004	3.163420E+8	5.061472E+12

You might discuss with the students the possibilities of extending the limits of land and water resources by technological advances that remove dependence on the land (synthetic food) or that create new sources of fresh water (desalinization of sea water). Point out that no new technology is spontaneous or without cost. The factories and raw materials that produce synthetic food, the equipment and energy to purify sea water must all come from the physical world system.\* Opening new land, farming the sea or expanding use of fertilizers and pesticides will require the use of nonrenewable resources like chemicals, fuels and metals (The Limits of Growth, p. 54). This will lead into the next section on nonrenewable resources.

---

\*It will be interesting to see if anyone suggests extraterrestrial sources here. You might then talk about the costs of retrieving materials from such sources.



## B. FOOD PRODUCTION/CAPITA

When we studied GNP, we found that the world is producing more goods and services each year, but the distribution of these goods and services is not uniform. While the GNP of some countries is rising, it is not keeping pace with the population rise of the country and hence the standard of living of the people of that country is not rising. In short, the people of many countries remain poor even though the GNP rises each year. What about food production and food production/capita? Is the world production of food rising each year? What effect does the food production in a country have on the people of that country?

Wheat production for various regions of the world are given in the table on the following page along with the population of that region over several years. Build a computer model of the growth in food production for each region and also for the food production/capita for each region.

### Questions

- (1) Is food production (wheat) increasing generally in all regions?
- (2) What effect does this have on the people in various regions? For example, how does the growth in food/capita for Africa compare with the growth in food/capita for Western Europe?

WHEAT PRODUCTION FOR REGIONS OF THE WORLD  
(THOUSANDS OF METRIC TONS)

Year	Africa	North America	South America	Asia	Europe	Oceania	U.S.S.R.	World
1970	7838	47988	8831	78803	66948	8177	99734	318319
1971	9012	60499	9719	85824	81408	8834	98760	354054
1972	9386	58272	10603	93561	81937	6823	85993	346575
1973	8715	64995	9966	88584	82319	12490	109784	376852
1974*	8465	64903	10653	83917	90482	11498	83913	359831

\*preliminary figures

Note: For population figures for regions, see Table 1 in the Population Growth section of these materials.

Source: Statistical Yearbook, 1975, pp. 154-155.

### C. NONRENEWABLE RESOURCES

#### 1. Chromium Reserves

So far, we have seen that exponential population growth has given rise to an exponential demand for food. The food supply is dependent upon land and fresh water and money invested in agriculture. Money invested in agriculture is in the form of machinery, pesticides and fertilizers, fuel, etc. Fuels, chemicals and metals are nonrenewable resources. Hence, the expansion of food production is linked to the availability of nonrenewable resources. In addition, the exponential growth of GNP is also dependent upon nonrenewable natural resources since the production of new goods requires more raw materials and fuels for manufacturing.

Give the students a table (they might get this from the almanac) showing some of the more important mineral and fuel resources which are the vital raw materials for today's major industrial processes. The table should contain the known global reserves of each resource and the present usage rate. Have the students write a computer program (model) which prints out the amount of the resource used each year and also the amount which remains unused. In this way, they can find how many years will elapse before the resource is used up. Build the model making the assumption that the current rate of usage will prevail.

Example:

Chromium, World's known reserves = 775 million metric tons.  
Amount mined annually = 1.85 million metric tons.

```
10 LET Y = 1970
20 LET R = 775
30 PRINT Y,R,1.85
40 LET Y = Y+1
50 LET R = R-1.85
60 IF R > 0 THEN 30
70 END
```

```
Y = YEARS
R = KNOWN RESERVES (million
metric tons)
1.85 = CONSUMPTION (million
metric tons)
```

Point out to the students that this is a linear relationship and that we have made the assumption that the current rate of usage will remain constant over the years. Now confront students with data from the 1971-1975 almanacs regarding chromium consumption. Have them extract the fact that chromium usage seems to grow exponentially. (The actual consumption of chromium is increasing at a rate of 2.6% annually.) Revise the model using this information.

```

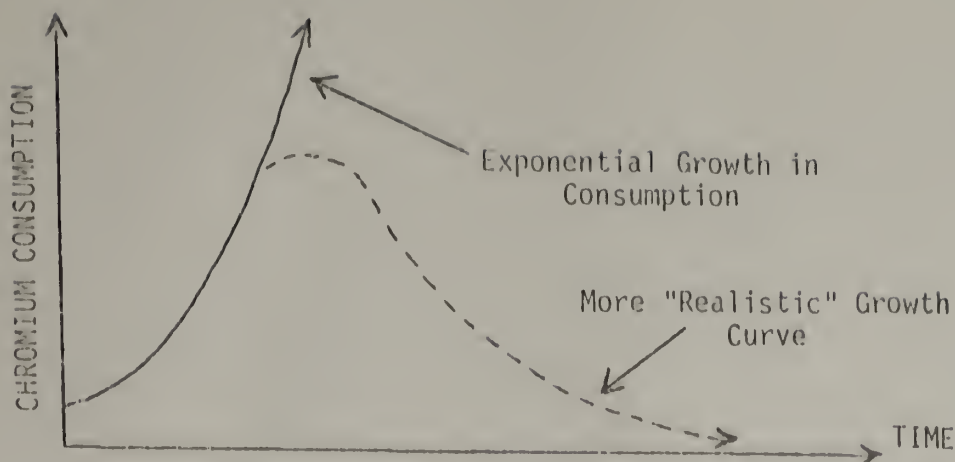
10 LET C = 1.85
20 LET Y = 1970
30 LET R = 775
40 PRINT Y,R,C
50 LET Y = Y+1
60 LET C = .026*C + C
70 LET R = R-C
80 IF R>0 THEN 40
90 END

```

Have the new model print out the chromium usage for the years 1970 to 1975 and now compare the model figures with those in the almanac. Discuss the "goodness of fit" of these figures with the known data as compared to the fit of the previous model.

Do not stop at this point and have the students believe that they have constructed a "correct" model. Point out the following: consumption of chromium might grow exponentially for the next several years because the price will remain low and constant due to new developments in mining technology which allow efficient use of lower and lower grade ores. But as the demand continues to increase, the advances in technology may not be fast enough to counteract the rising price of discovery, extraction, processing and distribution. The price of chromium would rise, slowly at first and then very rapidly. Higher prices would cause consumers to use chromium more efficiently and to substitute other metals wherever possible. (There is a parallel here to the recent rise in oil prices where consumers have begun to turn back thermostats to conserve and to substitute other forms of heat such as wood.) Eventually, the cost of mining the known reserves (by now only a fraction of the chromium is left) would be prohibitively high and mining of new supplies and usage would fall to zero (The Limits to Growth, pp. 64-65).

So rather than seeing consumption increase exponentially without let-up, a more realistic picture might be that consumption will increase exponentially for a time, level off and then begin to fall.



But again, stress the fact that we have here made another assumption. What assumption? Answer: That technology will not progress fast enough to keep up with demands and that these demands do not change. Also--and perhaps more important--we have neglected to mention the effect of recycling efforts. Perhaps a fair percentage of the 775 million metric tons is recoverable.

THE MODEL IS ONLY AS GOOD AS THE ASSUMPTIONS ON WHICH IT IS BASED!

## 2. Energy Consumption

In the following exercises, the students are asked to model the consumption of oil, coal, natural gas and uranium based on consumption rates extracted from almanacs.

Exercises:

- (1) Using the exponential chromium consumption model, make the assumption that a discovery in 1970 doubles the known reserves and run the model using this assumption. What is the effect of doubling the reserves (in the long run; in the short run)?
- (2) Build models for the consumption of oil, coal, natural gas and uranium based on the consumption rates provided and/or the growth in consumption rates.



Resource: Crude Petroleum

World Reserve (1974) = 75,530 Million Metric Tons

<u>Year</u>	<u>Production or Consumption</u>	
1970	2,265,800	Thousand Metric Tons
1971	2,400,250	Thousand Metric Tons
1972	2,531,520	Thousand Metric Tons
1973	2,775,520	Thousand Metric Tons
1974	2,792,080*	Thousand Metric Tons

\*Estimated or provisional figure  
(Round off to millions of metric tons)

Resource: Coal

World Reserves (1974) = 8,134,374 Million Metric Tons

<u>Year</u>	<u>Production or Consumption</u>	
1970	2,163,770	Thousand Metric Tons
1971	2,140,240	Thousand Metric Tons
1972	2,161,950	Thousand Metric Tons
1973	2,207,580	Thousand Metric Tons
1974	2,227,270*	Thousand Metric Tons

\*Estimated or provision figure

Resource: Natural Gas

World Reserve (1974) = 59,195 Billion Cubic Meters

<u>Year</u>	<u>Production or Consumption</u>	
1970	1,037,950	Million Cubic Meters
1971	1,110,970	Million Cubic Meters
1972	1,170,030	Million Cubic Meters
1973	1,231,320	Million Cubic Meters
1974	1,255,250*	Million Cubic Meters

\*Estimated or provision figure



Resource: Uranium

World Reserves (January 1975) = 1,080,500 Metric Tons  
(Not All Countries)

Year	Production or Consumption
1966	14,987 Metric Tons
1967	15,649 Metric Tons
1968	17,448 Metric Tons
1969	17,557 Metric Tons
1970	18,201 Metric Tons
1971	18,581 Metric Tons
1972	19,623 Metric Tons
1973	19,754 Metric Tons
1974	18,461* Metric Tons

\*Estimated or provisional figure

- (3) For further discussion, you might have students look up which nations have what percent of the total world resources and which nations use what percent of the total world resources. You might discuss the effects of shortages on the developing countries.
- (4) Another discussion which might result from the preceding exercises is that regarding the growth of pollution.

What happens to the chemicals, metals and fuels man has extracted from the earth after he has used and discarded them?

Their atoms are rearranged and dispersed into the soil, air and water of our planet. Our ecological system is able to absorb and reprocess many of these materials into substances which can be reused or are harmless to other forms of life. However, very often such large amounts of substances are released that it becomes impossible for natural absorption mechanisms to handle them. When this happens, the wastes of civilization become visible, annoying, and harmful. Hence, another exponentially increasing quantity in the world system is pollution (The Limits to Growth, p. 68).

THE TECHNOLOGICAL OPTIMIST

OR

"MALTHUS REBURIED"

UNIT THREE

- A. Technology -- Exponential Growth?
- B. The Effect of Technology on Food Supply
- C. The Effect of Technology on Population

Note to the Teacher:

The models constructed thus far have advanced a pessimistic, Malthusian outlook. This unit is designed to counterbalance this viewpoint with technological optimism and to reinforce the concept that all models are imperfect and only as good as their underlying assumptions.

So far, we have built models of population growth, economic growth, food production and nonrenewable resource usage. Our models have indicated that population and industrial output are growing exponentially while food production and nonrenewable resources are being consumed exponentially.

We have made the assumption in building our models that food production and natural resources are fixed and finite while the demand for these commodities is exponential. Our models have reflected these assumptions by indicating future shortages and eventual depletion of these resources.

The models we have built present a very unpleasant outlook for the future of mankind and our planet. We must remember, however, that the predictive power of the model depends to a large extent upon the validity of our initial assumptions. In this unit, we will make some very different assumptions based upon the growth of technology and look at the effect of these assumptions on our models.

#### A. TECHNOLOGY -- EXPONENTIAL GROWTH?

Technology-- the means by which man extends his power over his surroundings--is changing and changing much more rapidly than in the past. There are people alive now who were born before cars, airplanes, radio, television, plastics, drugs, computers and phonograph records became commonplace, perhaps even before they were invented. The speed with which new discoveries are turned into useful products is increasing as time goes by. As an illustration of this, consider the following table.

<u>Approximate Date Of Discovery</u>	<u>Invention</u>	<u>Time Taken From Discovery To Exploitation</u>
1831	Electric Motor	50 Years
1861	Telephone	15 Years
1887	Radio	7 Years
1925	Radar	9 Years
1925	Television	9 Years
1931	Atomic Reactor	11 Years
1938	Atomic Bomb	7 Years
1948	Transistor	5 Years
1958	Integrated Circuit	3 Years

(Taken from Technology, Man and the Environment, p. 21.)

Hence, the rate of change is itself accelerating which gives us reason to believe that technology may be another quantity which is increasing at an exponential rate.

## B. THE EFFECT OF TECHNOLOGY ON FOOD SUPPLY

When we examined the world's capacity to produce food, we made the assumptions that the most important factors which influenced production were the amount of arable land available and the fresh water resources. We also mentioned that other factors which affect production are the use of fertilizers, pesticides, farm machinery, etc., which can be lumped together under energy input. We should also consider that in addition to the nonrenewable resources of land, water, energy, chemical fertilizers, pesticides, etc., another important factor in food production is technology. Technology refers to agricultural science and includes genetic research in high yield grains, nitrogen fixation research, improved water and fertilizer management, new strategies for pest control, etc.

We made a small attempt to include technology in our food production model which we built earlier by suggesting that perhaps the land could be made more productive to feed more people. However, we made the assumption that perhaps the land would become twice as productive or four times as productive. The response of our model to these assumptions was simply to delay by a few years the time when all the available land would be used for food production. We might stop to consider now what the actual effect of technology is on productivity. Can it make the land twice or four times as productive? Or will productivity increase exponentially as time goes on? Consider the data below showing grain yields over time for the United States.

<u>Year</u>	<u>Corn Yield (bushels/acre)</u>
1870	27
1880	26
1890	26
1900	27
1910	26
1920	27
1930	25
1940	34
1950	42
1960	70
1970	85

(Source: These figures are approximations read from a graph in Population, Resources, and the Future, p. 61.)



Notice that from 1870 to about 1930 the yield was constant--about 26 bushels/acre. Hybrid corn was introduced in the early 1930's; by 1938, 15% of the corn planted was hybrid corn, and by 1948, 76% of the corn planted was hybrid. (Ecology, p. 610)

Using the data from 1930 on, construct a computer model of the yield and use your model to predict yields for 1930, 1990 and 2000. Compare the model values with the known values for 1930 to 1970 to get some indication of the "goodness" of the model.

Printout:

<u>Year</u>	<u>Actual Values</u>	<u>Predicted Values</u> (From Model)
1930	25	26
1940	34	36
1950	42	49
1960	70	67
1970	85	92
1980		126
1990		173
2000		237

Note that the original yield of 26 bushels/acre was almost doubled in 1950 to a value of 49 (according to the model) and quadrupled between 1970 and 1980. By the year 2000, the yield is almost 10 times the 1930 value. In fact, the yield is doubling about every 20 years. The model seems to indicate that productivity of the land can rise exponentially due to technology. How long this kind of growth could continue in reality is not known, but at least the model does indicate that the outlook for food production might be more optimistic than we originally thought when we viewed food production as depending solely on the amount of available arable land.\*

---

\*It should be noted that the introduction of hybrids was associated with the increased use of fertilizers and these two factors are responsible for the increase in U.S. corn yield. Hence, the increase cannot be attributed solely to technology, but to a combination of the use of technology and the use of a nonrenewable resource.

## Exercises

For additional examples of the impact of technology on land productivity, present the student with the data for dwarf wheat introduced in Mexico in the early 1950's and high-yielding rice strains in Taiwan.

- (1) Consider the data below for Mexican yields of dwarf wheat from 1950 to 1969. Build a computer model of increase or growth in yield.

<u>Year</u>	<u>Yield (pounds/acre)</u>
1950	800
1952	750
1954	1000
1956	1200
1958	1400
1960	1300
1962	1800
1964	2000
1966	2100
1968	2400

How many years elapse before the 800 pounds/acre yield is doubled according to your model?

- (2) Consider the data below for high-yield rice strains in Taiwan for the years 1950 to 1969. Build a computer model of the increase or growth in yield.

<u>Year</u>	<u>Yield (pounds/acre)</u>
1950	1500
1951	1580
1952	1640
1953	1780
1954	1840
1955	1780
1956	1900
1957	1950
1958	2020
1959	1960
1960	2100
1961	2150
1962	2220
1963	2400
1964	2450

<u>Year</u>	<u>Yield</u> (pounds/acre)
1965	2560
1966	2550
1967	2600
1968	2700

(Caution: This model may be linear and not exponential.)

### C. THE EFFECT OF TECHNOLOGY ON POPULATION

What happens to the population of a country as the level of technology rises? A rise in technology generally means that the country becomes more industrial and has more scientific skills which it can apply to manufacturing, medicine, transportation, etc. Industrialization brings about high levels of production and consumption. The standard of living of the people rises and so increasing technological development is associated with increasing socio-economic development. But again, what effect does all this have on the population growth?

Suppose we compare population growth in a sample of the developed countries (for instance, the United States, Canada and Sweden) with population growth in a sample of the developing countries (for instance, India and Ghana).

Since death rates in developed countries are uniformly low, we will concentrate on the birth rate. (The rate given in the following tables is actually fertility rate which is the number of births per 1000 females aged 10-49. Birth rate is the number of births per 1000 people in the population.)

Year	United States	Canada	Sweden	Japan
1945	2.6	3.1	2.6	Above 4.5
1950	3.1	3.5	2.4	4.0
1955	3.6	3.8	2.25	2.4
1960	3.7	3.9	2.25	2.0
1965	2.9	3.2	2.25	2.1
1970	2.4	2.3	1.9	2.1
1975	1.9	2.0	1.8	2.1

Year	India	Ghana
1963	3.6	5.0
1965	4.2	5.6
1973	6.1	6.7
1975	5.7	7.0

The figures show that the birth rates of developed countries have reached a peak (for the United States and Canada in the late 50's to early 60's) and are now dropping, whereas the developing countries continue to have high birth rates. Demographers (people who study populations) have suggested several reasons for this trend. One theory suggests that families have fewer children in the developed countries because the "cost" of raising a child increases as the family income increases. That is, in developed countries, like the United States, children are given more than the basic food and clothing requirements. They receive better housing and medical care, and education is both necessary and expensive. In the poorer developing nations, the cost of a child is very low. No additional living space is added to house a new child, little educational or medical care is available, clothing and food requirements are minimal. (The Limits to Growth, p. 114)

Whatever the reason for the drop in birth rate in the developed countries, the figures seem to indicate that population growth may not continue to increase without limit as we first thought in looking at population models. Population may eventually decrease and reach a stable level in the developed countries. This is encouraging since a population which levels off or stabilizes would mean less of a drain on that country's nonrenewable resources and food supply as well as less pollution.

Demographers also believe that as the developing countries become more industrialized, a transition occurs. First, death rates decrease due to better health care and more abundant food, then later birth rates begin to decrease as the standard of living continues to rise. This again sounds encouraging because it suggests that the developing

countries will parallel the population growth behavior that is now being observed in the developed countries and hopefully eventually tend toward stable populations. However, we should not become too optimistic for this reason: since the drop in birth rate lags the drop in death rate, there is rapid population growth for some time after the birth rate begins to decline. This further increase in the population size puts an added strain on the GNP/capita and impedes development. This leads demographers to fear that some underdeveloped countries may have already jeopardized their long-term welfare prospects due to their current population growth spurt.

This discussion leads us to conclude that predicting future population trends and sizes is very complicated. What is needed is additional data and better models to help get some idea of what will happen in the future.



## BIBLIOGRAPHY

- Abelson, P. H., ed. Food: Politics, Economics, Nutrition and Research. Washington, D. C.: American Association for the Advancement of Science, 1975.
- Bahr, H. M.; Chadwick, B. A.; and Thomas, D. L., eds. Population, Resources, and the Future: Non-Malthusian Perspectives. Provo, Utah: Brigham Young University Press, 1972.
- Cole, H. S., et al., eds. Models of Doom. New York: Universe Books, 1973.
- Dunn, S. L., and Wright, L. W. "Models of the U.S. Economy." The Mathematics Teacher 70 (1977): 102-110.
- Engineering Concepts Curriculum Project (ECCP). The Man-Made World. New York: McGraw-Hill Book Company, 1971.
- Hamilton, D. Technology, Man and the Environment. New York: Charles Scribner's Sons, 1973.
- Harte, J., and Socolow, R. H. Patient Earth. New York: Holt, Rinehart and Winston, Inc., 1974.
- The Human Population. San Francisco: W. H. Freeman and Company, 1974.
- Information Please Almanac Atlas and Yearbook 1965. New York: Simon and Schuster, 1964.
- Johnson, W. H., and Steere, W. C., eds. The Environmental Challenge. New York: Holt, Rinehart and Winston, Inc., 1974.
- Jolly, D. A. "Food Cost, Farm Policy and Nutrition." Journal of Nutrition Education 8(2), (1976): 56-58.
- Krebs, C. J. Ecology. New York: Harper and Row, Publishers, 1972.
- Meadows, D. H., et al. The Limits to Growth. New York: Universe Books, 1972.

The Official Associated Press Almanac 1974. Maplewood, New Jersey:  
Hammond Almanac, Inc., 1973.

Reader's Digest 1976 Almanac and Yearbook. Pleasantville, New York:  
The Reader's Digest Association, Inc., 1976.

Reed, C. B. Fuels, Minerals and Human Survival. Ann Arbor, Michigan:  
Ann Arbor Science Publishers, Inc., 1975.

United Nations. Statistical Yearbook 1975. New York, 1976.

The World Almanac 1965 and Book of Facts. New York: New York-  
Telegram and The Sun, 1965.

The World Almanac 1977 and Book of Facts. New York: Newspaper  
Enterprise Association, 1977.

APPENDIX E

DESCRIPTION OF SAMPLE STUDENTS

EXPERIMENTAL GROUP

Student Number	Age	Progressive Matrices		Modeling Achievement		APT Abstract Reasoning Subtest	WISC. IQ (est.)
		Pretest	Posttest	Pretest	Posttest		
1	12½-12¾	98	96	63	89	99	134
2	13-13	92	23	54	52	85	107
3	13-13	92	97	44	79	75	115
4	12½-12¾	86	75	35	74	97	112
5	13-13	80	80	38	70	85	102
6	12½-12¾	79	75	17	39	--	--*
7	13-13	75	75	32	53	70	107
8	12½-12¾	70	60	33	67	60	104
9	12½-12¾	70	43	41	67	85	114
10	12½-12¾	65	75	31	68	70	115
11	13-13	56	80	36	51	90	102
12	12½-13	55	69	34	34	75	102
13	12-12½	54	82	26	52	35	87
14	12½-12¾	50	55	16	33	30	85
15	13-13	50	47	24	24	50	98
16	13-13½	50	56	43	57	95	108

## EXPERIMENTAL GROUP--Continued

Student Number	Age	Progressive Matrices		Modeling Achievement		APT Abstract Reasoning Subtest	WISC. IQ (est.)
		Pretest	Posttest	Pretest	Posttest		
17	13½-13½	46	95	27	34	65	101
18	12½-12½	43	32	29	56	50	95
19	12½-12½	43	43	45	70	30	98
20	13-13	41	25	45	70	85	115
21	13-13	34	47	19	37	25	87
22	12½-12½	32	32	37	51	20	99
23	13-13	28	69	28	69	55	104
24	13-13	25	50	17	59	35	94
25	13½-14	20	29	23	39	40	87
26	14-14	18	24	2	25	3	76

\*APT score unavailable--transfer student

## CONTROL GROUP

Student Number	Age	Progressive Matrices		Modeling Achievement		APT Abstract Reasoning Subtest	WISC. IQ (est.)
		Pretest	Posttest	Pretest	Posttest		
1	13-13	96	96	63	76	99	128
2	13-13½	92	62	36	46	80	117
3	12-12½	86	75	47	56	65	106
4	13-13	85	56	38	55	85	121
5	12½-13	82	95	44	46	90	117
6	12½-12½	75	95	19	46	80	114
7	12½-13	75	56	28	36	80	110
8	12½-13	70	75	40	43	90	113
9	12½-12½	70	65	12	43	75	102
10	13-13	62	41	29	30	30	98
11	12½-13	60	62	27	37	90	100
12	13-13	56	75	39	34	95	110
13	12-12½	50	82	24	34	60	102
14	13-13½	50	32	24	8	60	94
15	13-13½	50	56	25	35	70	94
16	12½-12½	50	46	20	23	97	106



## CONTROL GROUP--Continued

Student Number	Age	Progressive Matrices		Modeling Achievement		Apt Abstract Reasoning Subtest	WISC. IQ (est.)
		Prefest	Posttest	Prefest	Posttest		
17	12½-13	46	62	41	40	90	126
18	12½-12½	43	79	10	25	55	96
19	12½-12½	43	50	25	21	70	102
20	12½-13	39	44	21	28	55	98
21	13½-14	32	50	1	12	85	106
22	12½-12½	32	50	31	38	80	102
23	13-13	31	50	1	23	15	84
24	14-14	25	50	11	20	15	68
25	12½-12½	23	18	20	32	10	88
26	13½-13½	15	23	18	17	15	79



